
The relationship between car following string instability and traffic oscillations in finite-sized platoons and its use in easing congestion via connected and automated vehicles with IDM based controller

Jie Sun^{a,b}, Zuduo Zheng^{b,*}, Jian Sun^a

^a Department of Traffic Engineering & Key Laboratory of Road and Traffic Engineering, Ministry of Education, Tongji University, 4800 Cao'an Road, Shanghai, 201804, China

^b School of Civil Engineering, The University of Queensland, St. Lucia 4072, Brisbane, Australia

A B S T R A C T

This paper focuses on two fundamental issues in traffic flow modelling: string stability of car following (CF), and oscillation of traffic flow. Its aim is to explore the complementary use of CF instability analysis and oscillation analysis to compress and ease traffic congestion in a connected environment. Each of these topics has been extensively investigated in the literature. However, each topic has been investigated separately and, despite the inherent conceptual and empirical closeness of the two concepts, little effort has been devoted to untangling their relationship.

To address this failing, we first define four types of oscillation – amplitude-decay oscillation, amplitude-ceiling oscillation, speed-deviation ceiling oscillation, and speed-deviation growth oscillation – and reveal their similarities and dissimilarities to CF instability. Based on the stability criterion, we then develop oscillation criteria to identify different types of oscillation by relaxing two unrealistic assumptions used in CF stability analysis (i.e., infinitely-long platoon and long-wavelength perturbation). Finally, to demonstrate how CF instability analysis and oscillation analysis can be combined to influence individual vehicle stability and improve traffic oscillations in a connected environment, a platoon of vehicles that experience an oscillation in the NGSIM data is used in a case study. In this case study, different control factors are used for different vehicle types: connected (but human-driven) vehicles, and automated vehicles.

Our analysis shows that a higher stability of some individual vehicles can alleviate the oscillation severity for the platoon. It also shows that desired time gap and maximum acceleration are two promising parameters that can be used to both improve individual vehicle stability and significantly smooth the oscillation of the platoon in a connected and/or automated environment. Of particular note, when considering all of the factors explored in our analysis, adjustment of the desired time gap is the most effective factor in smoothing traffic oscillations.

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* Corresponding author.

E-mail address: zuduo.zheng@uq.edu.au (Z. Zheng).

1. Introduction

For more than 60 years, since the early days of CF model development, stability of car following (CF) models have been studied in terms of the evolution of a small perturbation over time (i.e. local stability) and over a platoon (i.e. string stability) (Bando et al., 1998; Chandler et al., 1958; Ferrari, 1994; Herman et al., 1959; Lenz et al., 1999; Monteil et al., 2014, 2018; Ngoduy, 2013; Orosz et al., 2010a, 2011; Orosz et al., 2010b; Orosz et al., 2009; Sau et al., 2014, 2019; Sun et al., 2018; Swaroop and Hedrick, 1996; Talebpour and Mahmassani, 2016; Treiber and Kesting, 2013b), and significant progresses have been achieved. More specifically, several theoretical methods have been proposed for the local and string stability analysis of different types of CF models, and these are reviewed and compared in detail in our previous paper (Sun et al., 2018). Due to the fact that local stability can be easily achieved for most CF models, string stability is the concern of this paper. From here on, unless stated otherwise, ‘stability analysis’ refers to ‘string stability analysis’.

Meanwhile, traffic oscillation, a phenomenon commonly observed in congested traffic (Ahn et al., 2010; Bilbao-Ubillos, 2008; Chen et al., 2012; Laval and Leclercq, 2010; Zheng et al., 2010), can be interpreted as the manifestation of the instability of CF behaviour (Chandler et al., 1958; Orosz et al., 2010b; Wilson and Ward, 2011). However, the two fundamental issues in traffic flow modelling – stability of CF and oscillation of traffic flow – are often studied as separate issues in the literature. More specifically, string stability of CF is generally analysed from a theoretical perspective with many elaborated assumptions,¹ while traffic oscillation is commonly investigated from an empirical perspective using vehicular trajectories such as NGSIM data (NGSIM, 2006). They usually have different starting points and different end goals: the stability analysis of car following often starts from an equilibrium state with the end goal of deriving one or a set of boundary conditions that can theoretically divide the CF parameter plane into two regions: one is stable and one is not. So, analytical tractability is important for the stability analysis. On the other hand, the oscillation analysis of traffic often starts from a deceleration and acceleration cycle with the end goal of estimating traffic flow characteristics such as precursor of the oscillation, evolution of the oscillation, etc. So, empirical tractability is important for the oscillation analysis.

Nevertheless, significant progress has been made in the separate issues of stability analysis and oscillation analysis over recent decades. Specifically, stability criteria have been derived for many CF models. Unfortunately, however, it is generally difficult to directly use these stability criteria in designing traffic operation and control strategies because many assumptions adopted in conventional stability analysis are unrealistic in real traffic situations such as homogenous platoon (and are thus incapable of considering platoon heterogeneity and human driver heterogeneity); fixed equilibrium state; arbitrarily long platoon; and long-wavelength instability. On the other hand, while many oscillation studies reported in the literature put a great amount of effort into scrutinizing empirical data, identifying patterns, and postulating the mechanisms for the formation and propagation of oscillations, their analytical rigorousness needs further improvement.

Despite the inherent and intertwined nature of stability analysis and oscillation analysis, and the obvious appeal of blending them in a complementary manner, results of stability analysis (e.g. the stability criteria of a CF model) cannot be directly used in oscillation analysis for two reasons: 1) their unrealistic assumptions (the linear instability and growing oscillations are equivalent when the assumptions are the same for the two types of analysis); and 2) the fact that traffic oscillation and CF stability are usually defined differently (e.g. the amplitude of oscillation is often measured by the largest speed drop during deceleration and acceleration, which is independent on the equilibrium state, while the magnitude of the disturbance in stability analysis is measured by using the absolute deviation from the equilibrium state [e.g. either speed deviation or acceleration/deceleration rate], which means that the magnitude is dependent on the equilibrium state). These issues also arise in a few studies that attempt to understand the origin and propagation of traffic oscillation from the perspective of instability (more discussion on this matter is presented in the next section). The clear relationship between such differences and contributing factors (i.e. platoon size and duration of perturbation) is not reported in previous studies, which is one motivation of this study. In summary, there is a great need to link stability analysis and oscillation analysis.

To create this link, we first classify oscillations into four types: amplitude-decay oscillation; amplitude-ceiling oscillation; speed-deviation ceiling oscillation; and speed-deviation growth oscillation (a detailed definition of each oscillations type is presented later). A natural way of bridging the stability and oscillation is to modify the well-established stability criteria for CF models by relaxing some of the assumptions adopted in conventional stability analysis, while the infinitely long platoon and long-wavelength perturbation are the focus of this study, as they can easily cause significant differences between CF instability (from a linear stability analysis) and traffic oscillation in real traffic. This will enable the development of concrete criteria for identifying different oscillation types. It then becomes possible to use stability analysis to directly understand and control the formation and propagation of empirical oscillations in real traffic.

Therefore, a natural question to then ask is how we might utilize both CF instability analysis and oscillation analysis complementarily for the purpose of compressing and easing traffic congestion – the ultimate goal of this study. Specifically, the advent of connected and automated vehicles can facilitate the application of stability analysis to oscillation mitigation, as a connected environment enables researchers/operators to influence (or even guide) individual drivers’ behaviours in order to achieve desirable traffic dynamics. Thus, in the last part of this study, we evaluate the impact of CF stability on oscillation characteristics by controlling certain parameters of CF models. For this purpose, one empirical oscillation extracted from the

¹ There are some experiment-based stability studies (Sugiyama et al., 2008; Nakayama et al., 2009); however, the majority of the stability studies are analytical, and they are referred to as conventional stability analysis in this paper.

NGSIM data is used. Moreover, because the control factors that are suitable for one vehicle type are not necessarily effective for another, different vehicle types present different opportunities and challenges for traffic control. Thus, we also analyse different control factors for different vehicle types, specifically, connected (but human-driven) vehicles and automated vehicles.

The remainder of this paper is organized as follows: Section 2 reviews the notable endeavours of CF string stability analysis and traffic oscillation in the literature; Section 3 scrutinizes the similarities and dissimilarities between CF instability and traffic oscillation, and develops the theoretical criteria for identifying different types of traffic oscillations; Section 4 explores the impact of CF stability changes on oscillation characteristics, and Section 5 summarizes and discusses the main findings of this study.

2. Literature review

As two fundamental concepts in traffic flow modelling, CF stability and traffic oscillations have been extensively investigated for many decades, and significant progress has been made. This section reviews the major findings of these previous investigations of the two concepts.

2.1. Stability analysis

Stability analysis has been conducted since the early days of CF model development (Chandler et al., 1958; Herman et al., 1959). From the theoretical perspective of CF models (Sun et al., 2018), it mainly focuses on how the small disturbance of a leading vehicle evolves over time and space. Two types of stability are usually studied: local linear stability, and string linear stability. As mentioned in the introduction, string linear stability – which focuses on the stability of a platoon of vehicles over space under the influence of a small perturbation originating from the first vehicle – is the concern of this study. This is because of its close relationship with traffic oscillation, and the easy satisfaction of local stability criterion. It is string stable if the perturbation strictly diminishes over space/vehicles; otherwise, it is string unstable. (For more information on different types of stability, see Sun et al. (2018), Treiber and Kesting (2013b).

To implement the conventional stability analysis theoretically, several assumptions are usually adopted. These assumptions are: vehicles are identical, and the number of vehicles in the platoon should be arbitrarily large (i.e. an arbitrarily long and homogeneous platoon); long-wavelength instability always occurs first (which means that the wavelength of the first unstable perturbation tends to infinity); and the perturbation of the leading vehicle is a steady periodical oscillation. These assumptions are not realistic in real traffic, and significantly affect the applicability of the stability analysis. The impact of several important factors (e.g. the number of vehicles, and the magnitude, intensity, and duration of perturbation) on the applicability of the stability analysis was assessed in Sun et al. (2018). Several studies considered heterogeneous platoon (e.g. Ngoduy, 2013; Talebpour and Mahmassani, 2016), and real traffic oscillation (e.g. Li and Ouyang, 2011; Treiber and Kesting, 2011) in stability analysis. These studies bring more insight for the understanding of traffic oscillations. However, the close relationship between the traffic oscillation and CF instability has yet to be unravelled, as the platoon size and perturbation duration considered in stability analysis are usually unrealistic, which is the motivation of this study.

Regarding different types of CF models (i.e. basic, time-delayed, and multi-anticipative/cooperative CF models, as defined in Sun et al. (2018)), several notable methods borrowed from control theory were used to analyse CF stability. Specifically, the direct transfer function based method, which calculates the magnitude of transfer function with the Fourier ansatz, was implemented in CF modelling by Chandler et al. (1958) and Herman et al. (1959) with basic and time-delayed CF models. Many researchers also applied the Laplace transform based method for analysing the CF stability by using Laplace transform to calculate the magnitude of transfer function in the frequency domain (Ferrari, 1994; Ge and Orosz, 2014; Liu et al., 2001; Monteil et al., 2018; Orosz et al., 2011, 2010b; Swaroop, 1997; Swaroop and Hedrick, 1996; Wilson and Ward, 2011). As a general approach in stability analysis, the characteristic equation-based method is used. This method is based on the characteristic equation induced from linearized CF models with the use of exponential ansatz to represent perturbations, and its use is widely reported in the literature (Lenz et al., 1999; Monteil et al., 2014; Ngoduy, 2013, 2015; Orosz et al., 2010a, 2011; Orosz et al., 2010b; Sau et al., 2014, 2019; Treiber and Kesting, 2013b). The mathematical details of these methods are provided in Sun et al. (2018).

According to these methods, there is a general string stability criterion for basic CF models (without considering any time delays):

$$\frac{1}{2} - \frac{f_{\Delta v}}{f_v} - \frac{f_s}{f_v^2} > 0,$$

where $f_s = \frac{\partial f}{\partial s}|_e$, $f_v = \frac{\partial f}{\partial v}|_e$, $f_{\Delta v} = \frac{\partial f}{\partial \Delta v}|_e$ are the Taylor expansion coefficients of the acceleration function at the steady state in terms of different variables after first-order linearization.

This criterion is used in this study as the base for developing criteria to identify different oscillation types.

2.2. Oscillation analysis

As a typical phenomenon observed in traffic flow, traffic oscillation has been studied for the almost 50 years since it was first observed in the Lincoln Tunnel (Edie, 1961). To identify traffic oscillation and measure its characteristics, second-

order difference of cumulative data method in time domain was adopted in [Ahn and Cassidy \(2007\)](#) and [Ahn et al. \(2010\)](#). Meanwhile, [Li et al. \(2010\)](#) and [Zheng et al. \(2011a, 2011b\)](#) proposed short time Fourier transform and wavelet transform, respectively, to analyse oscillation characteristics. It is shown that the wavelet transform is particularly effective in identifying traffic oscillation and measuring its characteristics. Thus, wavelet transform is also adopted in this paper.

Researchers are particularly interested in understanding the mechanism of traffic oscillation (e.g. its origin and propagation). Recently, much empirical evidence indicates that traffic oscillations on multi-lane freeways form and develop primarily due to lane changing manoeuvres ([Ahn and Cassidy, 2007](#); [Kerner and Rehborn, 1996](#); [Mauch and Cassidy, 2002](#)). In particular, [Mauch and Cassidy \(2002\)](#) found that the impact of lane-changing behaviours on oscillations is different for different traffic conditions. [Ahn and Cassidy \(2007\)](#) used loop data and vehicle trajectory data to analyse the formation and growth of several traffic oscillations, and confirmed that lane changing behaviour is the main trigger of these oscillations.

Meanwhile, many researchers pointed out that the formation and propagation of traffic oscillations can also be primarily caused by drivers' discrepant CF behaviours. Furthermore, using trajectory data, [Zheng et al. \(2011b\)](#) found that oscillations can originate in either lane changing or CF behaviour, and that oscillation can have a regressive effect on the latter. Thus, many CF models that aim to reproduce the characteristics of traffic oscillations are proposed in the literature. For instance, models with probabilistic time headway and reaction time were adopted by [del Castillo \(2001\)](#) and [Kim and Zhang \(2008\)](#) to reproduce traffic oscillations. Meanwhile, [Yeo and Skabardonis \(2009\)](#) proposed an asymmetric driving theory, and conjectured that traffic oscillations occur at the deceleration process in congested traffic.

[Laval and Leclercq \(2010\)](#) extended Newell's simple CF model ([Newell, 2002](#)) to describe the formation and propagation of stop-and-go waves, by explicitly accommodating the different characteristics of individual drivers (e.g. those with aggressive or timid driver behaviour). Their simulation results indicate that their model could satisfactorily reproduce some typical features of traffic oscillations that were observed in the empirical data (e.g. oscillation period and amplitude). On the basis of [Laval and Leclercq \(2010\)](#) model, in turn, [Chen et al. \(2012\)](#) presented a behavioural CF model that considers different driver characteristics before and during the traffic oscillations. The relationship between rubbernecking behaviour and the features of oscillations was developed, and was validated using simulation. [Saifuzzaman et al. \(2017\)](#) incorporated a driver's task difficulty profile in CF models to capture driver behaviour changes in response to the evolution of traffic oscillation. The formation and propagation of the stop-and-go traffic oscillations were found to be closely related to the task difficulty level.

As well as this progresses in gathering empirical evidence related to characteristics of traffic oscillations, a few studies attempted to understand the origin and propagation of traffic oscillation from the perspective of the instability in the CF process, by analysing the stability properties of certain CF models. More specifically, the propagation of oscillation is attributed to the over-reactions (i.e. instability) of drivers in the CF process ([Chandler et al., 1958](#); [Herman et al., 1959](#)). [Orosz et al. \(2009\)](#) investigated the stability of a CF model with driver's reaction time delay. As a result of this investigation, they defined a mechanism in which traffic flow is stable but sensitive to external disturbances, and this sensitivity results in the formation of oscillations. [Orosz et al. \(2010b\)](#) also attempted to understand traffic jam formation and propagation through the stability of a CF model. [Li and Ouyang \(2011\)](#) proposed a describing function-based approach to quantify traffic oscillation. Later, [Li et al. \(2014\)](#) used this approach to analyse the oscillation properties of a nonlinear CF model adapted from Newell's model ([Newell, 1961](#)). They found that the instability of the CF model is important in generating some reasonable oscillation growth. [Treiber and Kesting \(2018\)](#) incorporated external noise and action points into the intelligent driver model (IDM, [Treiber et al., 2000](#)), and analytically calculated the instability of the new model. The results show that external noise and action points deteriorate the stability of IDM, and this instability leads to traffic oscillation on freeways. Although traffic oscillation was linked with CF instability in these studies, the unrealistic assumptions commonly adopted in conventional stability analysis were also present.

In summary, the literature shows that stability analysis and oscillation analysis have been commonly studied as distinct entities, and that the close relationship between the two has yet to be unravelled. More importantly, the way to use stability analysis to directly understand and control the formation and propagation of empirical oscillations in real traffic, and the way to utilize both CF instability analysis and oscillation analysis complementarily for the purpose of compressing and smoothing traffic congestion, have not yet been explored. These research needs motivate this study.

3. Traffic oscillation types and how to identify them based on CF instability criterion

For the convenience of discussion, we consider a typical scenario of CF, where identical vehicles with length l are travelling in a single lane, without overtaking. Vehicle n follows vehicle $(n-1)$; x_n and x_{n-1} denote the position of the n th and $(n-1)$ th vehicle, respectively; v_n and v_{n-1} denote the speed of the n th and $(n-1)$ th vehicle, respectively; $s_n = x_n - x_{n-1} - l$ is the gap between these two vehicles (the rear bumper of the $(n-1)$ th vehicle to the front bumper of the n th vehicle); $\Delta v_n(t) = v_{n-1}(t) - v_n(t)$ is the relative speed of vehicle n with the leading vehicle $(n-1)$. In a steady state, all vehicles travel at the same speed v_e (equilibrium speed) with the same gap s_e (equilibrium gap).

As already mentioned, although traffic oscillation and CF instability are two closely related concepts in traffic flow theory, they have been studied in isolation, and a clear relationship between them has yet to be understood. Therefore, in this section, we scrutinize the similarities and dissimilarities between CF instability and traffic oscillation, and link them by relaxing the assumptions made in conventional stability analysis. To this end, we first categorize traffic oscillation into four different types, and then compare them with CF instability. Based on this comparison, we clearly establish the link between each oscillation type and CF instability; moreover, we propose criteria for identifying each oscillation type by modifying the

stability criterion. These criteria not only formally reconcile these two important concepts, but also point the way to the use of stability analysis in managing traffic oscillations.

3.1. The definition of four types of oscillation

Conventional stability analysis only considers two regions: the stable and the unstable. Furthermore, the condition for stability is strict. As per Sun et al. (2018); Treiber and Kesting (2013b), to be string stable the perturbation (i.e. the spacing/speed deviation from the equilibrium spacing/speed that is used as the indicator) needs to strictly attenuate for each leader-follower pair as it propagates along the platoon, and is formulated as:

$$\|\varepsilon_1\|_\infty > \|\varepsilon_2\|_\infty > \dots > \|\varepsilon_k\|_\infty > \dots > \|\varepsilon_n\|_\infty \quad (1)$$

where $\|\varepsilon_k\|_\infty = \max_t |\varepsilon_k|$ is the maximum magnitude of the k th vehicle's perturbation within infinite time.

Meanwhile, the growth of traffic oscillation can be measured with two indicators: 1) the largest speed drop during the deceleration and acceleration period (as this indicator is a parameter that has been frequently used to describe characteristics of oscillation (Zheng et al., 2011a,b; Chen et al., 2012; Saifuzzaman et al., 2017), and more specifically to measure the oscillation amplitude (Zheng et al., 2011b); and 2) the lowest speed of the platoon (as this indicator illustrates the severity of congestion: If the lowest speed of the followers is above the lowest speed of the leader, it means the congestion is getting better; otherwise, congestion is getting worse as it propagates backwards). Using these two indicators, we categorize traffic oscillation into four types according to the oscillation amplitude evolution and speed deviation while the perturbation from the first vehicle propagates upstream in the platoon (similar to the upstream convective instability as observed in real traffic (Treiber and Kesting, 2013b)). The definition of each oscillation type is given below. In these definitions, we consider a homogenous platoon that contains n vehicles. To better interpret the definitions, the speed drop of the k th vehicle in i th cycle of oscillation is defined as $v_{drop}^{k,i} = v_{dec}^{k,i} - v_{acc}^{k,i}$, the speed deviation of the k th vehicle in the i th cycle of oscillation from the equilibrium is defined as $v_{dev}^{k,i} = v_e^k - v_{acc}^{k,i}$, where $v_{dec}^{k,i}$, $v_{acc}^{k,i}$ is the speed at the starting point of the deceleration and acceleration phases of the k th vehicle in the i th cycle of oscillation, respectively; and v_e is the equilibrium speed. To simplify notation, and without causing confusion, vehicle number k is dropped from the notation hereafter (unless otherwise stated).

- Amplitude decay oscillation (ADO, Type I): The oscillation amplitude strictly decays along the platoon, and is formulated as:

$$\|\delta_1\|_\infty > \|\delta_2\|_\infty > \dots > \|\delta_k\|_\infty > \dots > \|\delta_n\|_\infty \quad (2)$$

where $\|\delta_k\|_\infty = \max_t v_{drop}$ is the maximum speed drop of k th vehicle within infinite time.

- Amplitude ceiling oscillation (ACO, Type II): The amplitude does not always decay. While this means that Eq. (2) does not hold, the oscillation amplitudes of following vehicles are not larger than (i.e. ceiled by) the amplitude of the first vehicle, and is formulated as:

$$\text{Eq.(2) does not hold and } \|\delta_1\|_\infty \geq \max \|\delta_k\|_\infty, \quad k \in [2, 3, \dots, n]. \quad (3)$$

- Speed-deviation ceiling oscillation (SCO, Type III): The maximum oscillation amplitude of following vehicles exceeds the amplitude of the first vehicle, but the speed deviations of the following vehicles are constrained by the speed deviation of the first vehicle, and is formulated as:

$$\|\delta_1\|_\infty < \max \|\delta_k\|_\infty \text{ and } \|\sigma_1\|_\infty \geq \max \|\sigma_k\|_\infty, \quad k \in [2, 3, \dots, n] \quad (4)$$

where $\|\sigma_k\|_\infty = \max_t v_{dev}$ is the maximum speed deviation of k th vehicle within infinite time.

- Speed-deviation growth oscillation (SGO, Type IV): The maximum speed deviation of the following vehicles exceeds the speed deviation of the first vehicle, and is formulated as:

$$\|\sigma_1\|_\infty < \max \|\sigma_k\|_\infty, \quad k \in [2, 3, \dots, n]. \quad (5)$$

The severity of these four types of oscillation increases with each type (i.e. from Type I to Type IV). In order to effectively manage traffic congestion, it is critical to detect oscillation type at an early stage, and to implement operational and control strategies accordingly. Unlike stability analysis in which traffic is roughly divided into two regions – the stable and the unstable – the categorization of traffic oscillations into these four types enables us to develop and implement more appropriate operational and control strategies tailored to oscillation characteristics and severity.

By comparing stability analysis and these four oscillation types, we realize that, with the exception of the Type I oscillation (which is similar to the stable region), the oscillation types (i.e. Types II-IV) cannot be further distinguished through a stability analysis. This is because, in such an analysis, they will simply be regarded as unstable – and on the basis of many unrealistic assumptions (e.g. arbitrarily long platoon, long-wavelength instability, and steady oscillation). This is a main reason why conventional stability analysis has been criticized as being over-simplified, unrealistic and thus of limited practical value (Sun et al., 2018; Wilson and Ward, 2011). In real traffic, there are various types of oscillation: Type II and Type III oscillations are commonly observed in real traffic, and are the focus of traffic control and management. Furthermore, distinctions between Type III and Type IV oscillations are critical in daily traffic operations: If these two types of oscillation

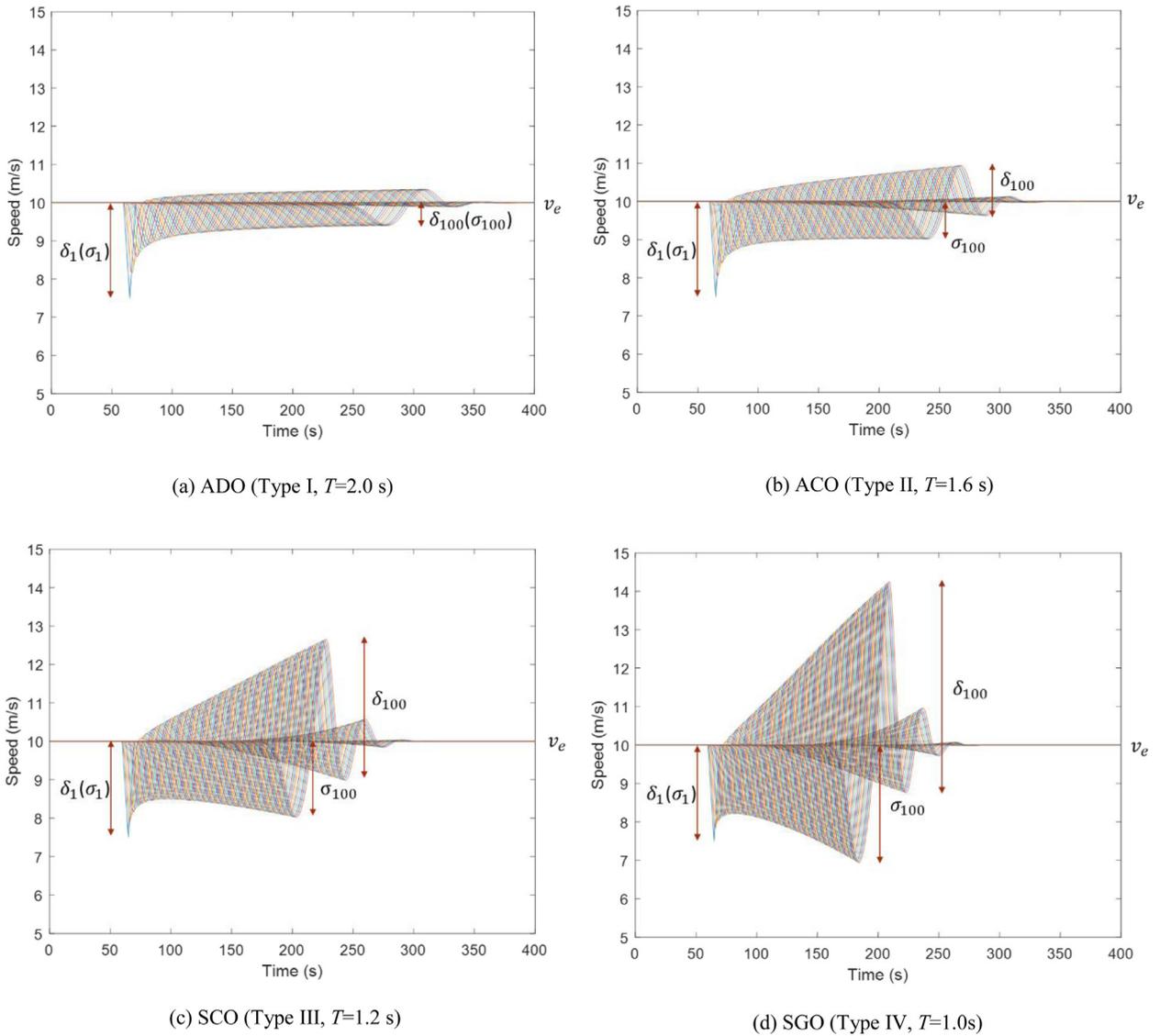


Fig. 1. Time-speed diagrams for different types of oscillations (the same simulation scenario as for Fig. 3. $\alpha = 1$ m/s²).

cannot be identified in a timely manner and promptly controlled, they will soon cause the congestion to spread to other parts of the network.

Although it is difficult to use empirical observations to directly verify the existence and the properties of different oscillation types (the same issue with the stability analysis) as the oscillation types are defined under some assumptions (e.g. homogenous platoon, steady-state equilibrium), four time-speed diagrams using simulations (the simulation setups are introduced in detail in Section 3.2.2) for illustrating different types of oscillation are presented, with different parameter combinations as shown in Fig. 1. Each type of oscillation could be reproduced by selecting different combinations of parameters.

Furthermore, we have calculated the flow and density for each case using Edie's definition (Edie, 1963). As shown in Fig. 2, all the points fall into the congestion branch of the fundamental diagram while the congestion level increases from Type I to Type IV. While Type IV oscillation is obviously different with others, the other three types of oscillations are more difficult to be differentiated from each other by visually checking the plot. By plotting the linear trend line for Type I to Type III oscillation, the data points of Type I, II and III all have a linear relationship with high goodness-of-fit. The three trend lines clearly have different coefficients and intercepts, which convincingly demonstrates that each of these three oscillation types corresponds to a different traffic flow level. This observation is consistent with the widely reported scattering phenomenon in the flow-density diagram (Cassidy and Mauch, 2001; Cassidy et al., 2011). Thus, classifying oscillation into these four types is reasonable as each type corresponds to a different traffic flow level and traffic characteristics reflected in these oscillation types are consistent with empirical observations.

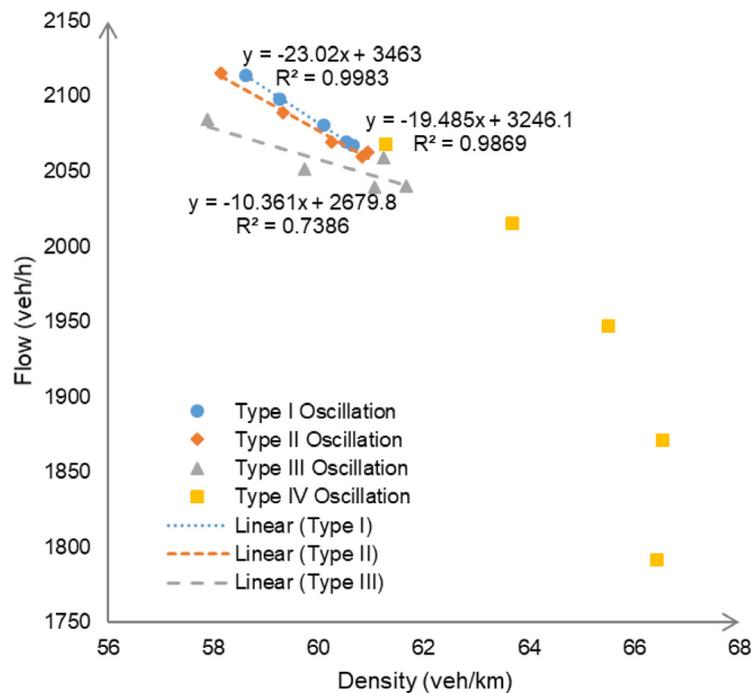


Fig. 2. Flow-density diagram for different oscillation types.

Note that the four oscillation types do not always exist with every initial perturbation (e.g. dipole-like perturbation). However, whether all four types of oscillation can be generated from a particular initial perturbation is not important because the purpose of categorizing oscillation into four types is to enable us to timely detect an oscillation type and then develop and implement more appropriate operational and control strategies. If a certain type of oscillation cannot be observed from a particular initial perturbation, it would simplify the control task, i.e., no need to consider these missing oscillation types.

While oscillations have attracted much attention in the recent literature, most of the studies are totally empirical, and there is a lack of solid analytical evidence to guide the development of efficient operational and control strategies. More specifically, no criterion in empirical oscillation analysis (i.e., similar to the stability criterion in stability analysis) has been developed to detect and classify different oscillation types. Thus, this study is motivated by the need to find a way to effectively integrate the theoretical rigour of stability analysis and the practical importance of oscillation analysis. This theoretical and practical integration will, in turn, provide the basis for the development of concrete congestion alleviation strategies. The categorization of oscillation (as defined above) makes it feasible to directly link the theoretical results of stability analysis to traffic oscillation analysis for two reasons: the inherent similarity of the two concepts; and the fact that the main cause of dissimilarities between traffic oscillation and CF instability are assumptions that do not apply to real traffic situations. More specifically, the stability criterion can be modified and used to identify each oscillation type, as elaborated below.

3.2. Identification of different types of oscillation

We aim to develop concrete criteria for identifying different oscillation types by modifying the well-established stability criterion for CF models. A natural way to approach this is by relaxing some of the (unrealistic) assumptions adopted in stability analysis – assumptions that are the main cause of dissimilarities between traffic oscillation analysis and CF instability analysis. More specifically, the number of vehicles in platoon, and the duration of disturbance (related to the wavelength of disturbance), are factors that may influence the modification of the criterion. First, the platoon size in real traffic naturally varies significantly, and is closely related to traffic demand, road capacity, driving behaviour, traffic conditions, etc. In any case, the platoon size in real traffic is impossible to become or approach to be infinite. Moreover, the stability analysis adopts the long-wavelength instability assumption (which means that the wavelength of the first unstable perturbation is close to be infinite). The long-wavelength instability assumption clearly contradicts the commonly observed and reported duration (about a few seconds) of the initial perturbation that leads to a growing oscillation in real traffic. Therefore, the assumptions relevant to these two factors in stability analysis are relaxed. We first determine the relationship between the modification and the two factors, based on the simulation results. The mathematical form of the modification is then developed, calibrated, and evaluated.

3.2.1. A general form of oscillation criteria

According to Sun et al. (2018), there is a general string stability criterion for basic CF models (without considering any time delays), as shown in Eq. (6):

$$\frac{1}{2} - \frac{f_{\Delta v}}{f_v} - \frac{f_s}{f_v^2} > 0 \tag{6}$$

where $f_s = \frac{\partial f}{\partial s}|_e$, $f_v = \frac{\partial f}{\partial v}|_e$, $f_{\Delta v} = \frac{\partial f}{\partial \Delta v}|_e$ are the Taylor expansion coefficients of the acceleration function at the steady state in terms of different variables after first-order linearization.

As the stability criterion of basic CF models (without considering any time delays) shown in Eq. (6), we define $S = \frac{1}{2} - \frac{f_{\Delta v}}{f_v} - \frac{f_s}{f_v^2}$. If S is positive, the CF model is stable; otherwise, it is unstable.

Considering that different types of oscillation are the manifestation of different degrees of CF instability, a modification factor k_i ($i \in [1, 2, 3]$) is added to the stability criterion to capture the impact of different types of oscillation, as shown in Eq. (7):

$$O_i = S + k_i \tag{7}$$

where O_i denotes the severity of oscillation, and determines the boundary between Type i oscillation and Type $i + 1$ oscillation. $O_i > 0$ means that it is a Type i oscillation. It is worth noting that, for one oscillation, it falls into the less severe type of oscillation first. For example, if O_1 and O_2 are both positive, it is a Type I oscillation; if O_1 and O_2 are both negative and O_3 is positive, it is a Type III oscillation; when O_1, O_2, O_3 are all negative, it is a Type IV oscillation. According to the severity of different types of oscillation, there is a relationship that:

$$O_1 \geq O_2 \geq O_3 \tag{8}$$

Since the number of vehicles in the platoon n and the duration of disturbance t_d may cause the difference between CF instability and traffic oscillation, it is reasonable to assume that the modification factor k_i is a function of these two factors, as shown in Eq. (9):

$$k_i = f_i(t_d, n) \tag{9}$$

3.2.2. Relationship between the modification factor, platoon size, and disturbance duration

With a general form of oscillation criterion, as shown in Eq. (7), an important task is to specify a function that can reasonably represent the relationship between the modification factor k_i and two factors (i.e. platoon size n and disturbance duration t_d). Numerical simulations are deployed to achieve this goal. Using this approach, we aim to demonstrate how a linkage between the stability analysis and oscillation analysis can be established by modifying the well-established stability criterion. The factor k_i is the critical component of this empirical approach in developing the oscillation criteria with IDM. With other CF models and perturbations, a similar procedure could be easily applied to establish such link.

(1) The simulation setup

Following the setup in Sun et al. (2018), a platoon of identical vehicles enter a very long, single-lane straight road and quickly reach the equilibrium state, driving at the same speed (10 m/s or 36 km/h) and corresponding equilibrium gap. At $t = 60$ s, a disturbance is introduced to the first leading vehicle by forcing it to first decelerate and then accelerate for the same time duration. After experiencing the disturbance, vehicles converge to the original equilibrium state.

For the purpose of illustration, and while other CF models can be applied by following the same process, IDM (Treiber et al., 2000) is used in this study. There are two main reasons for this: i) IDM is widely used in the literature, and is capable of satisfactorily reproducing many characteristics of traffic flow, in particular, the stop-and-go oscillation characteristic; and ii) the model's stability is also analysed in the literature (Ngoduy, 2013, 2015; Treiber and Kesting, 2013b).

IDM is mathematically formulated in Eqs. (10) and (11):

$$\dot{v}_n(t) = \alpha \left[1 - \left(\frac{v_n(t)}{v_0} \right)^4 - \left(\frac{s_n^*(t)}{s_n(t)} \right)^2 \right] \tag{10}$$

$$s_n^*(t) = s_0 + T v_n(t) - \frac{v_n(t) \Delta v_n(t)}{2\sqrt{\alpha\beta}} \tag{11}$$

where v_0 is the desired speed; $s_n^*(t)$ is the desired gap; s_0 is the minimum gap in the standstill situation; T is the desired time gap; α is the maximum acceleration; and β is the comfortable deceleration.

IDM's stability criterion can be constructed by using its Taylor expansion coefficients, as shown in Eqs. (12)–(14) according to this string stability criterion:

$$f_s = \frac{2\alpha}{s_e} \left(\frac{s_0 + T v_e}{s_e} \right)^2 \tag{12}$$

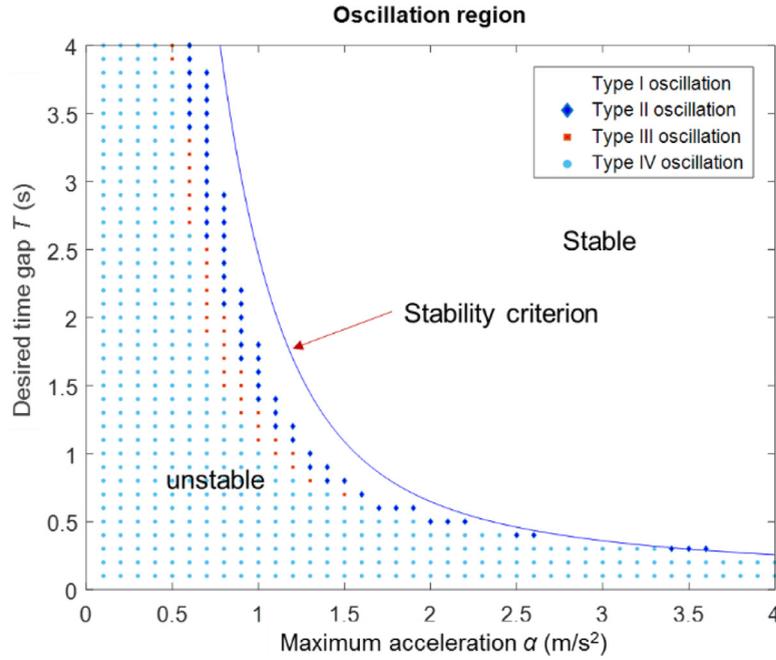


Fig. 3. The stability and oscillation regions for IDM. (The solid curve denote the boundary between the theoretical stable area and the unstable area; the points denote different types of oscillation from simulation; the number of vehicles in the platoon is 100; the deceleration duration is 5 s and the deceleration is 0.5 m/s²).

$$f_v = -\alpha \left[\frac{4}{v_0} \left(\frac{v_e}{v_0} \right)^3 + \frac{2T(s_0 + Tv_e)}{s_e^2} \right] \quad (13)$$

$$f_{\Delta v} = \sqrt{\frac{\alpha}{\beta} \frac{v_e s_0 + Tv_e}{s_e}} \quad (14)$$

where v_e and s_e denote equilibrium speed and equilibrium spacing in homogenous traffic, respectively.

In each simulation step, the following vehicles move forward by using the acceleration $a_n(t + \Delta T) = \dot{v}_n(t)$ according to IDM, while the speed and position of vehicles are updated with the trapezoidal rule, as shown in Eqs. (15) and (16) (Treiber and Kanagaraj, 2015). The speed drops, and speed deviations from the equilibrium speed of vehicles in the platoon are then collected for determining the oscillation types:

$$v_n(t + \Delta T) = v_n(t) + \frac{1}{2}(a_n(t) + a_n(t + \Delta T))\Delta T \quad (15)$$

$$x_n(t + \Delta T) = x_n(t) + v_n(t)\Delta T + \frac{a_n(t)(\Delta T)^2}{2} \quad (16)$$

where ΔT is the updated time step, and is set as 0.1 s in our simulations.

In the simulations, the number of vehicles in the platoon (where the range considered is 20–100, with a step size of 20) and the deceleration duration (which is measured by deceleration time, and the range considered is 2–10 s with a step size of 1 s) are two main varying factors. The intensity of the disturbance (measured by deceleration) is also changed for each deceleration duration with a range of 0.1–0.5 m/s² and a step size of 0.1 m/s². The total speed decrease is constrained to less than 5 m/s to ensure a small disturbance; this constraint ensures the validity of a linear stability analysis. Two hundred and twenty-five scenarios are considered. To show the differences between stability regions and oscillation regions, the stability and oscillation regions governed by the desired time gap T (0.1–4 s) and maximum acceleration α (0.1–4 m/s²) are plotted for each combination of number of vehicles and deceleration duration. More specifically, 1600 combinations of different parameters of IDM are simulated. Other parameters of IDM are default: desired speed $v_0 = 120$ km/h; comfortable deceleration $\beta = 1.5$ m/s²; minimum gap $s_0 = 2$ m; and vehicle length $l = 5$ m. For each simulation run, the platoon's oscillation type is assessed using the definition of oscillation types (see Section 3.1).

Based on the oscillation assessment results for all the simulation runs, we plot the oscillation region of IDM. One example is shown in Fig. 3. Different types of dots represent the simulation runs in which the platoon was in a different type of oscillation, while the blank area represents the simulation runs in which the platoon is a Type I oscillation. This figure clearly demonstrates the differences between the stability region and different types of oscillation. It also shows that Type II, III, and IV oscillations cover almost the whole unstable region, albeit with a small inconsistency. In addition, although

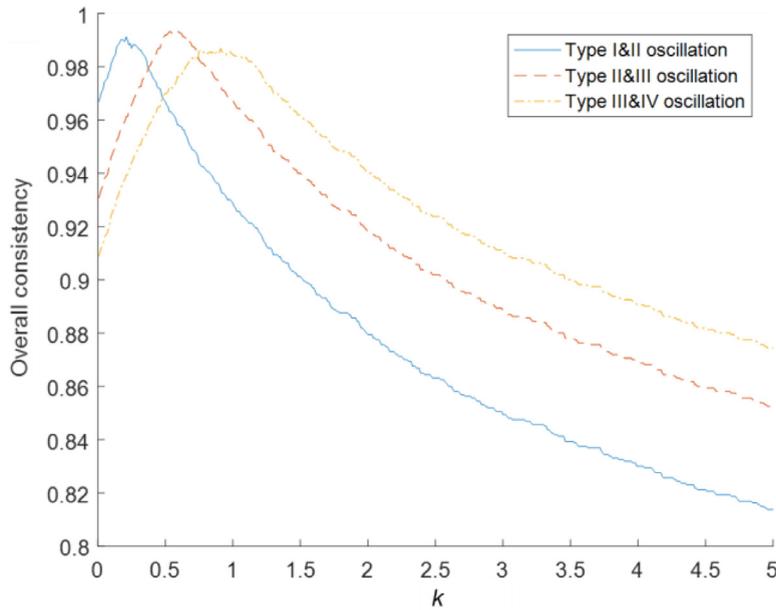


Fig. 4. Overall consistency (%) vs. k_i for boundaries of different types of oscillation (the same simulation scenario as for Fig. 3).

frequency of the Type IV oscillation is much higher than that of Type II or III, there is a considerable difference between the stability boundary and the Type III and IV oscillation boundary. This difference could be used for more effective control in practice.

(2) Analysis

With the simulation results of different combinations of number of vehicles and deceleration duration, the value of k_i for the boundaries between different types of oscillation can be determined in a way that maximises the overall consistency (i.e. the proportion of consistent simulation runs with the theoretical results to the number of total simulation runs) between the simulation result and the theoretical result (using Eq. (7)). An example is shown in Fig. 4. This figure shows that with the increase of k_i , the overall consistency between the theoretical results and the simulation results for different types of oscillation first increases, and then decreases. For each oscillation type, the overall consistency curve is convex; thus, there is a global maximum (higher than 98%). The value of k_i corresponding to this global maximum is considered as the optimal value for each scenario, and is used to identify its relationship with the platoon size and deceleration duration. Note that in each scenario, a more severe type of oscillation is expected to have a larger value of k_i .

The relationships between k_i ($i = 1, 2, 3$) and deceleration duration t_d for $n = 60$ are shown in Fig. 5 (the complete results for different numbers of vehicles are presented in Figs. A1–A3). Note that each deceleration duration is related to five decelerations in the simulation; thus, five points are plotted for one t_d in Fig. 5 (there is an overlapping of points in the figures). With a fixed platoon size, the value of k_i decreases with the increase of deceleration duration. This pattern is consistent across all oscillation types. From an examination of these figures, it is obvious that a logarithm function can be used to approximate the relationship between k_i and t_d . Admittedly, the logarithmic relationship is not as pronounced for k_1 , particularly when the number of vehicles is relatively large. This exception is largely caused by the relative insensitivity of the overall consistency to a small range of k_1 (especially when k_1 is small), as shown by the solid curve in Fig. 4. For k_1 around 0.4, the overall consistency trend is not strictly convex, and there are small but frequent fluctuations. Overall, nevertheless, it is a reasonable assumption that the relationship between k_i and t_d is approximately logarithmic.

Similarly, the relationship between k_i and the number of vehicles n is illustrated in Fig. 6. To save space, only the relationship between k_i and n for $t_d=5$ s is plotted here. As is the case for the relationship between k_i and t_d , there is also an obvious logarithmic relationship between each k_i and n when t_d is fixed.

As illustrated in Figs. 5–6, there is a logarithmic relationship between k_i and number of vehicles n , and between k_i and deceleration duration t_d , respectively. Inspired by this observation, we update Eqs. (9) to (17) to mathematically define the relationship between k_i and these two factors.

$$k_i = a_1 \ln\left(\frac{a_2}{n} + 1\right) \times \ln\left(\frac{a_3}{t_d} + 1\right) \tag{17}$$

where a_1, a_2, a_3 are parameters that need to be calibrated. Since k_i decreases with the increase in the number of vehicles and deceleration duration, n and t_d are denominators in the natural logarithmic function. Additionally, we add a constant term $- '1'$ to each logarithmic term to ensure the positivity of k_i . When n and t_d approach infinity, the identification of oscillations collapses to the conventional stability analysis (i.e., $k_i \approx 0$).

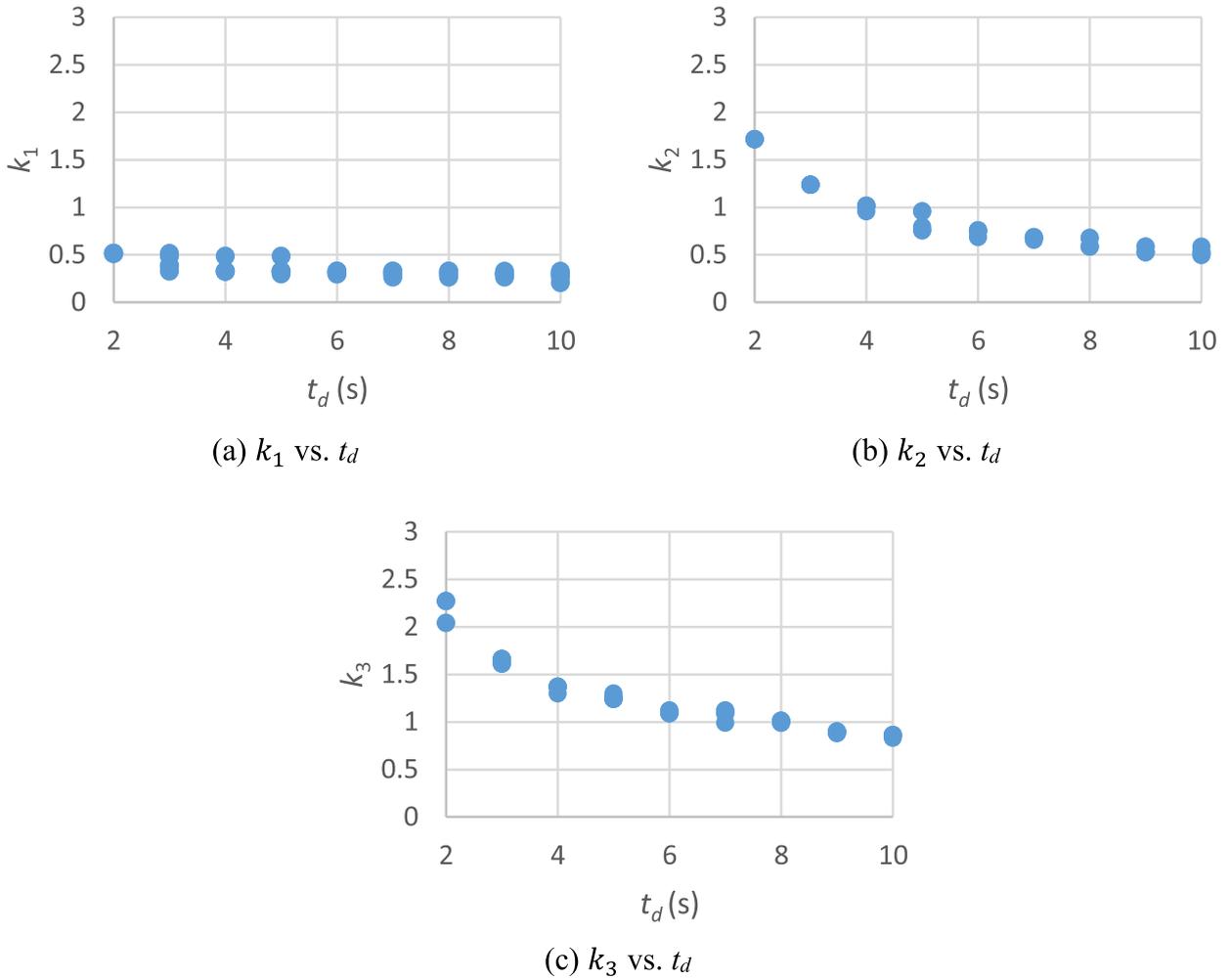


Fig. 5. The relationship between k_i and t_d for $n = 60$.

We then use the Curve Fitting Toolbox in MATLAB (MathWorks, 2001) to calibrate the three parameters for the boundaries between different types of oscillation. The results are shown in Table 1. As indicated by the high values (i.e. close to, or above 0.90) of *R-squared* in the table, a good fitting has been obtained for the boundary between different oscillation types. This further confirms the soundness of the proposed functional form of k_i shown in Eq. (17).

With the calibrated parameters, we then test the performance of the proposed oscillation criteria by evaluating the overall consistency between the theoretical results (by combining Eq. (7) and (17)) and the simulation results. The results for the three boundaries between the four types of oscillation are shown in Fig. 7, where each point denotes the consistency result of a single scenario (as earlier explained, there are 225 scenarios). The results are grouped by the number of vehicles (where $n = 20, 40, 60, 80,$ and 100). In each group, the deceleration duration increases gradually from 2 s to 10 s. This figure clearly reveals that for the majority of the scenarios, the consistency between the results from the proposed oscillation criteria and those from the simulation is greater than 97%. This demonstrates the excellent performance of the proposed oscillation criteria. However, when the deceleration duration is very small (e.g. 2 s), the consistency between the results from the proposed oscillation criteria and those from the simulation is relatively weaker, especially for scenarios with a small number of vehicles (20 or smaller).

Table 1
Calibration results of oscillation criteria.

	a_1	a_2	a_3	<i>R-squared</i>
Type 1&2 oscillation	0.43	97.21	18.13	0.95
Type 2&3 oscillation	23.79	3.84	4.51	0.90
Type 3&4 oscillation	253.70	6.57	0.37	0.89

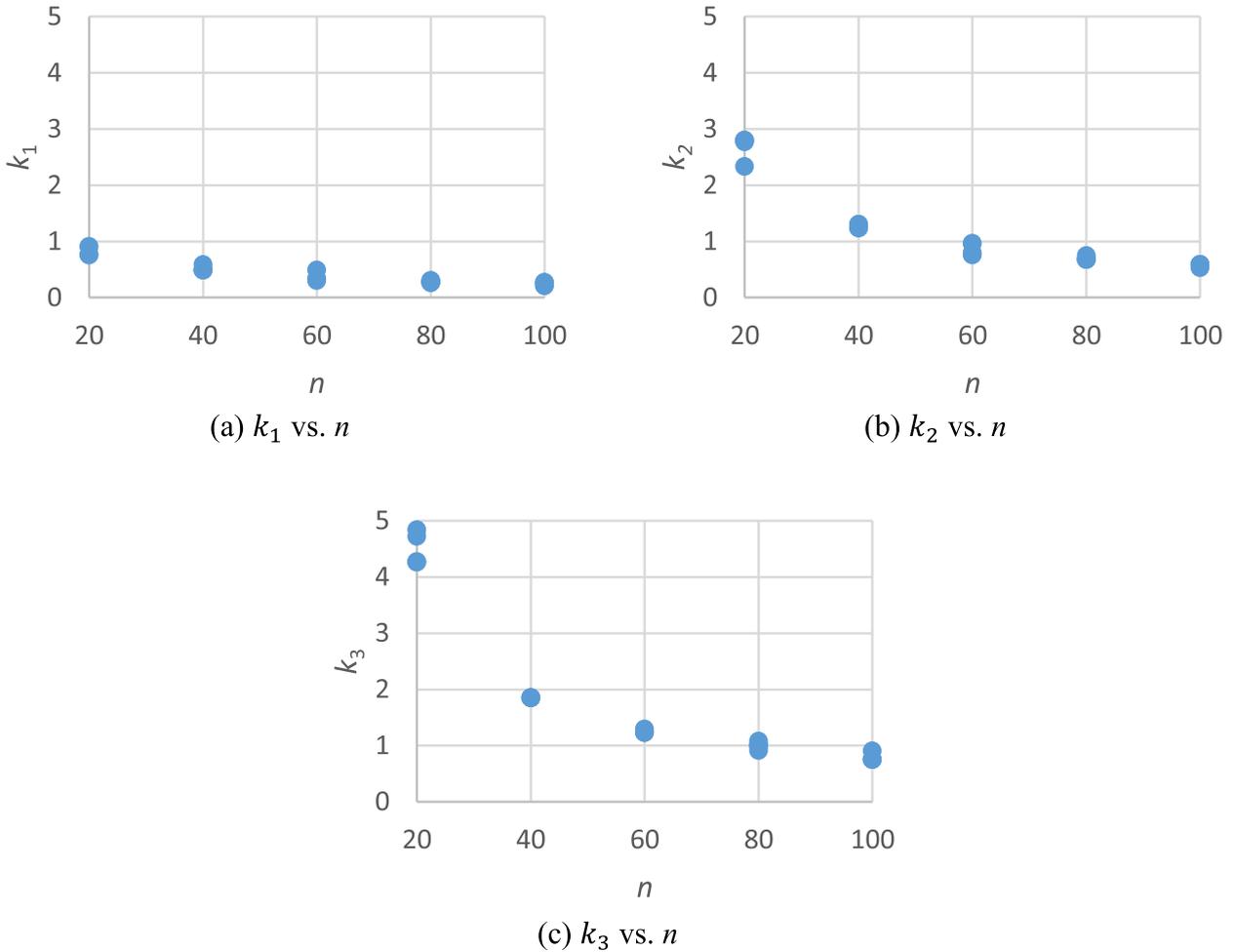


Fig. 6. The relationship between k_i and n for $t_d=5$ s.

4. Application: the combined use of CF instability analysis and oscillation analysis in easing traffic congestion

So far, we have clearly defined the similarities and dissimilarities between CF instability and traffic oscillation, categorized oscillation into four distinctive types, and developed the criteria to identify different oscillation types based on the stability criterion. A natural question to ask next is: How does change in CF instability impact traffic oscillation? This is a question with important practical value: Such knowledge enables us to utilize both CF instability analysis and oscillation analysis complementarily for the purpose of compressing and easing traffic congestion. This is the ultimate goal of this study, and the specific task of this section of the paper.

To begin with, although the simulation of string stability usually uses a homogeneous platoon of vehicles with identical parameters, in real-world traffic, vehicles in a platoon are most likely to be heterogeneous. Meanwhile, it is each vehicle's inherent role to directly cope with the propagation of disturbance from its preceding vehicle (i.e. to amplify/diminish the disturbance magnitude from the preceding vehicle and to propagate the "modified" disturbance to the next vehicle). Obviously, each vehicle's ability to cope with the disturbance matters, and has a direct impact on how an oscillation evolves over time and space. Thus, we can use stability analysis to obtain a stability profile for each vehicle in a heterogeneous platoon, and to then investigate how changes in an individual vehicle's stability profile impact oscillation evolution (in particular, its severity). The ability to influence individual driving behaviour will become feasible once connected and automated vehicles are the main vehicles on the road.

To this end, the observed trajectory data containing notable oscillations are used to calibrate IDM for the analysis of its stability. With the calibrated IDM, we produce the oscillation by introducing a small disturbance to the first vehicle in the platoon, and then evaluate the characteristics of the resulting oscillation experienced by the platoon. More importantly, we explore the impact of IDM's stability change on the oscillation characteristics.

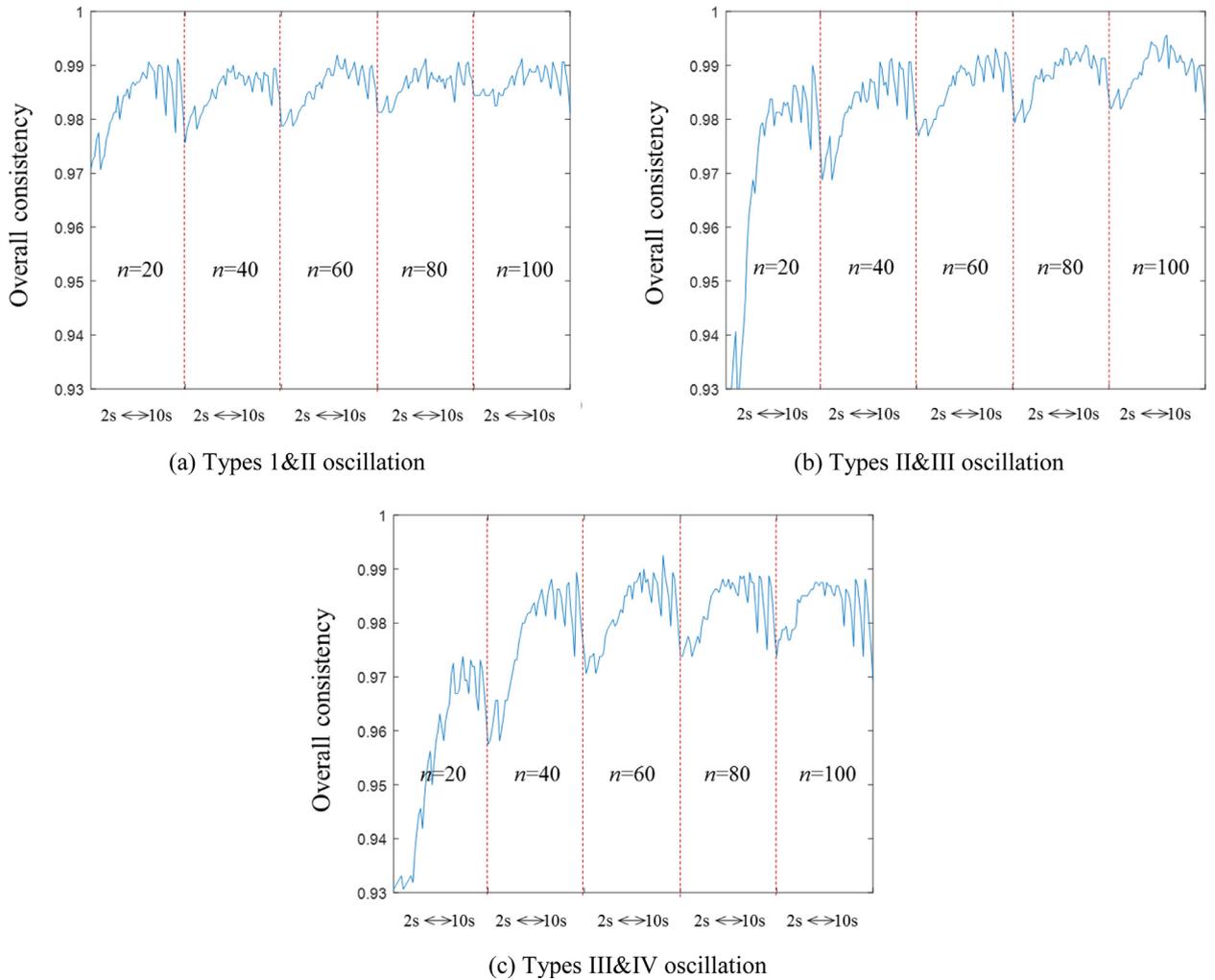


Fig. 7. Validation results for the oscillation criteria (grouped by number of vehicles, deceleration duration increases gradually from 2 s to 10 s in each group).

4.1. Data preparation

As per previous studies (Chen et al., 2012; Saifuzzaman et al., 2017; Zheng et al., 2011b, 2013), the trajectory data in Lane 1 of the southbound US-101 segment in Los Angeles from 7:50 a.m. to 8:05 a.m. (collected by the Next-Generation Simulation project (NGSIM, 2006), are used in this study. In this data set, traffic oscillations appear every 2–3 min, and are caused by (some) drivers' rubbernecking the clean-up work being undertaken in the median strip (Chen et al., 2012). The vehicle trajectories in Lane 1 are shown in Fig. 8, where the oscillation analysed in this study is also depicted. From this oscillation, a platoon of 65 vehicles is selected for further analysis; these vehicles did not change lanes in the selected time window.

Significant noise in the NGSIM data is reported in the literature (Coifman and Li, 2017; Punzo et al., 2011, 2012; Treiber and Kesting, 2013a). Thus, a multi-step process, as recommended by Montanino and Punzo (2013), is undertaken to smooth the acceleration and speed profiles before any further analysis is conducted.

4.2. Calibration

We first calibrate IDM, using the 65 vehicles selected above. With the calibrated parameters, we can assess the stability of IDM with each set of parameters, using IDM's theoretical stability criterion.

Model calibration aims to find optimal values for unknown parameters, with the objective of minimizing the differences between observed variables (e.g. spacing and speed extracted from real trajectories) and simulated variables (corresponding variables from simulated trajectories) (Kesting and Treiber, 2008; Ciuffo et al., 2013; da Rocha et al., 2015). Two calibration methods are considered here: 1) one parameter set is calibrated for each trajectory pair (this enables us to analyse the

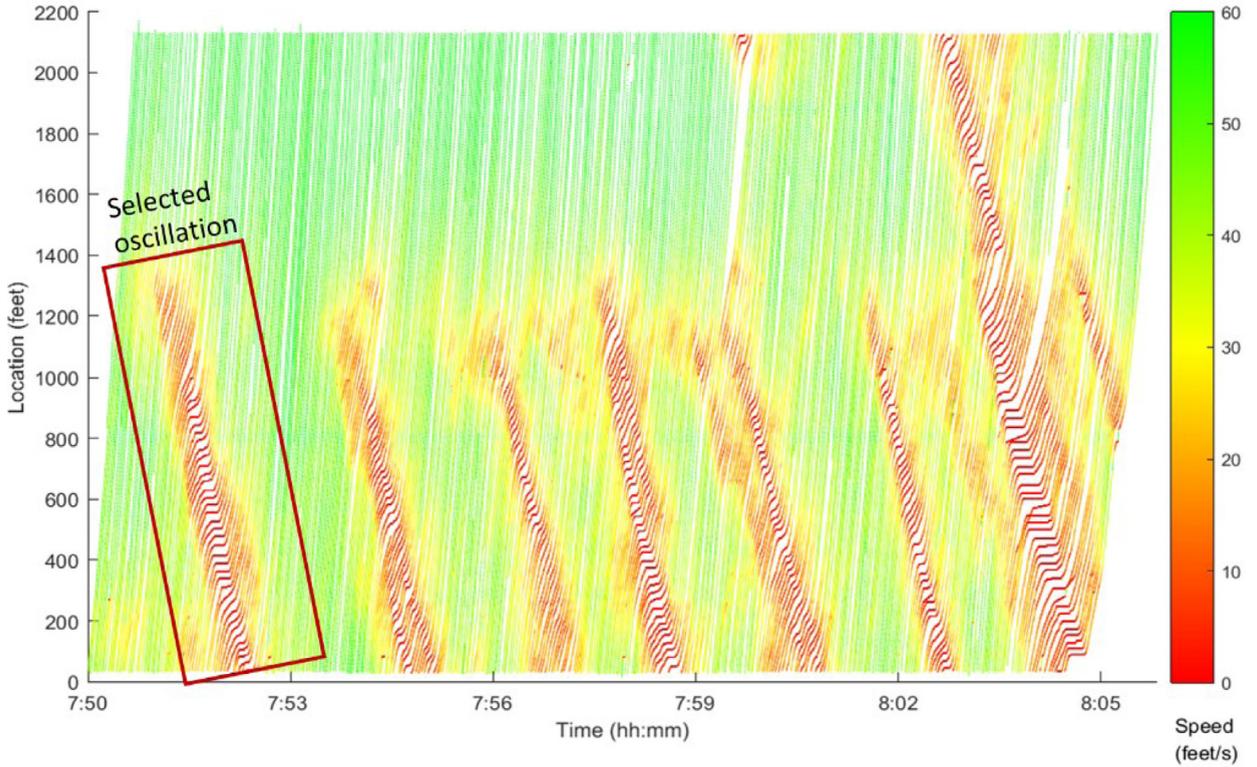


Fig. 8. Illustration of vehicle trajectories (Lane 1 of the US 101 study site).

behaviour of individual drivers); and 2) a single parameter set is calibrated for the whole platoon, and each simulated trajectory in the platoon is based on the real trajectory of the preceding vehicle. This method can facilitate the analysis of the homogenous platoon.

Genetic algorithm (GA) has been widely used for calibrating CF models in previous studies (Kesting and Treiber, 2008; Saifuzzaman et al., 2015; Sun et al., 2018). Due to its advantages in global optimization (Powell, 1973), and its supremacy over other optimization methods (Punzo et al., 2012), it is also employed in this study to estimate the parameters of IDM. As per the literature (Kesting and Treiber, 2008; Punzo and Montanino, 2016; Sharma et al., 2019a), the difference between simulated and observed spacing is an appropriate indicator to measure the calibration performance, as it can simultaneously reduce the speed and acceleration estimation error. More specifically, in terms of spacing, the root mean squared normalized error (RMSNE) is adopted as the objective function of GA, as shown below:

$$RMSNE = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\frac{S_i^{sim} - S_i^{obs}}{S_i^{obs}} \right)^2} \tag{18}$$

where i denotes the observation id; S_i^{sim} is the i th simulated spacing; S_i^{obs} is the i th observed (real) spacing; and N is the total number of observations.

We run GA with the Optimization Toolbox in MATLAB (MathWorks, 2012) to calculate the optimal parameters of IDM. In this optimization process, each parameter estimate is subject to the range specified in Table 2. The process is repeated 10 times, and the set of parameters with the minimum RMSNE is selected.

Table 2
Calibration results of IDM.

Parameter	Range	One parameter set for one vehicle				One parameter set for platoon
		Average	Median	Min	Max	
RMSNE	–	0.12	0.12	0.0019	0.39	0.33
Desired speed v_0 (km/h)	[1.0, 150]	95.27	86.69	44.41	133.23	73.51
Desired time gap T (s)	[0.1, 4.0]	0.99	0.82	0.20	2.86	1.34
Minimum gap s_0 (m)	[1.0, 10]	4.34	3.98	1.23	9.25	3.82
Maximum acceleration α (m/s ²)	[0.1, 4.0]	1.91	1.38	0.49	4.0	1.04
Comfortable deceleration β (m/s ²)	[0.1, 4.0]	2.29	2.19	0.31	4.0	1.71

A summary of the calibration results from the two calibration methods is also presented in Table 2. The relatively small RMSNE from the first calibration method, and the fact that the calibrated parameter values are largely consistent with typical values in real highway traffic (Treiber and Kesting, 2013b), indicate the effective performance of the calibration process.

When calibrating a CF model, rich variations and sufficient number of vehicles are two important factors in avoiding overfitting or obtaining unrealistic parameter values. Based on the comprehensive analysis using both numerical experiments and field observations, Sharma et al. (2019a) has revealed that for IDM, the parameters can be estimated with low calibration errors using less complete trajectories. Although the NGSIM data do not contain free traffic (however, the data contain nearly free traffic/very light congestion traffic), to obtain a reasonable estimate of v_0 , if IDM is used, then the presence of exact free flow is not required, although desirable. As shown in Table 2, the calibrated v_0 has a mean of 95 km/h, with a median and a maximum of 87 km/h and 133 km/h, respectively. Such a distribution is not unrealistic, and is simply a consequence of driver heterogeneity. In contrast, the calibrated v_0 for the one parameter set per platoon scenario is indeed unrealistically low (i.e., 74 km/h). This implies that in reproducing traffic oscillations of a platoon with heterogeneous vehicles, IDM should be calibrated for each pair of vehicles instead of for the platoon as a whole.

With the calibrated IDM, the stability of a vehicle is calculated using $S = \frac{1}{2} - \frac{f_{\Delta v}}{f_v} - \frac{f_s}{f_v^2}$. (As earlier mentioned, $S > 0$ means that it is theoretically string stable.) The stabilities of 65 vehicles calibrated with the first calibration method are plotted in Fig. 9. From Fig. 9, we observe that most vehicles' stability values are around 0, while a few vehicles have fairly small stability values (much smaller than 0). The vehicles with negative stability values can easily cause unstable driving behaviour of the following vehicles in the platoon. In addition, most unstable vehicles distribute in the middle of the platoon (Vehicles 19–34). The stability of the calibrated CF model with the second calibration method (one parameter set for the whole platoon) is -0.5492 ; this is in the unstable region.

4.3. Analysis of oscillation characteristics

Before we use the calibrated IDM to investigate the impact of stability change on traffic oscillations, it is important to check whether the calibrated model is capable of reproducing the typical oscillation characteristics that are observed in the real world. Note that the true replication of the observed oscillations is not the purpose of this analysis. Such replication is an unrealistic goal, as the model is not calibrated for this task. Rather, we focus on the capability of the calibrated model to generate the typical trends and features of observed traffic oscillations.

To this end, after identifying the starting point of the deceleration and acceleration phases using wavelet transform method (Saifuzzaman et al., 2017; Zheng et al., 2011a,b), the oscillation amplitudes of the selected vehicles in NGSIM data are calculated. As shown in Fig. 10, the amplitude increases slowly at first, and then begins to surge to about 15 m/s after Vehicle 18. It then stays at a relatively high level, thus indicating a well-developed oscillation. The growth trend of this oscillation is consistent with the stability evolution of these vehicles (as shown in Fig. 9). This finding further demonstrates the close relationship between oscillation growth and CF instability.

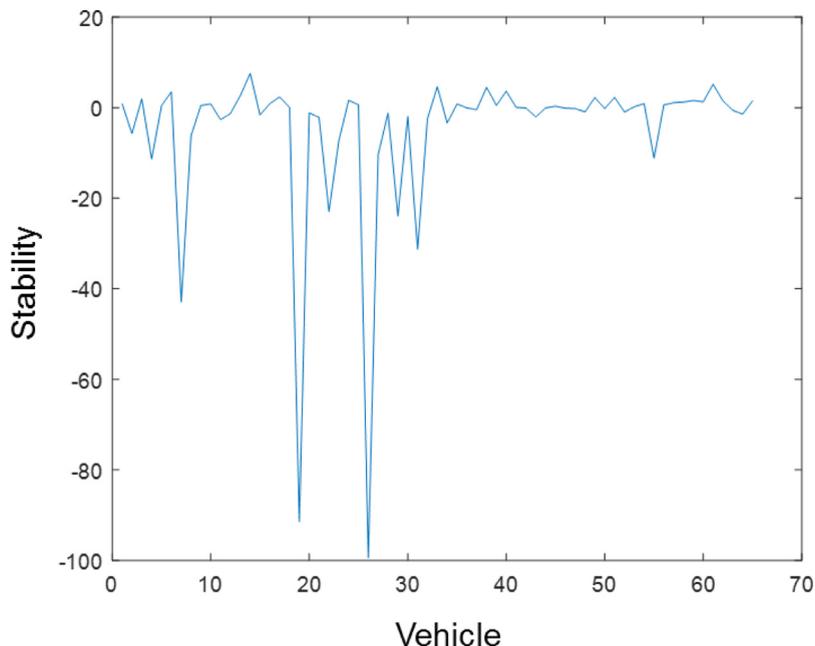


Fig. 9. Stability of calibrated parameter sets.

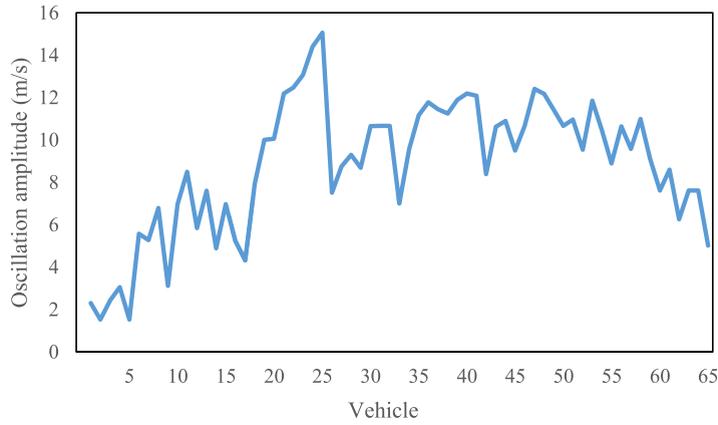


Fig. 10. Oscillation amplitudes of vehicles in NGSIM data.

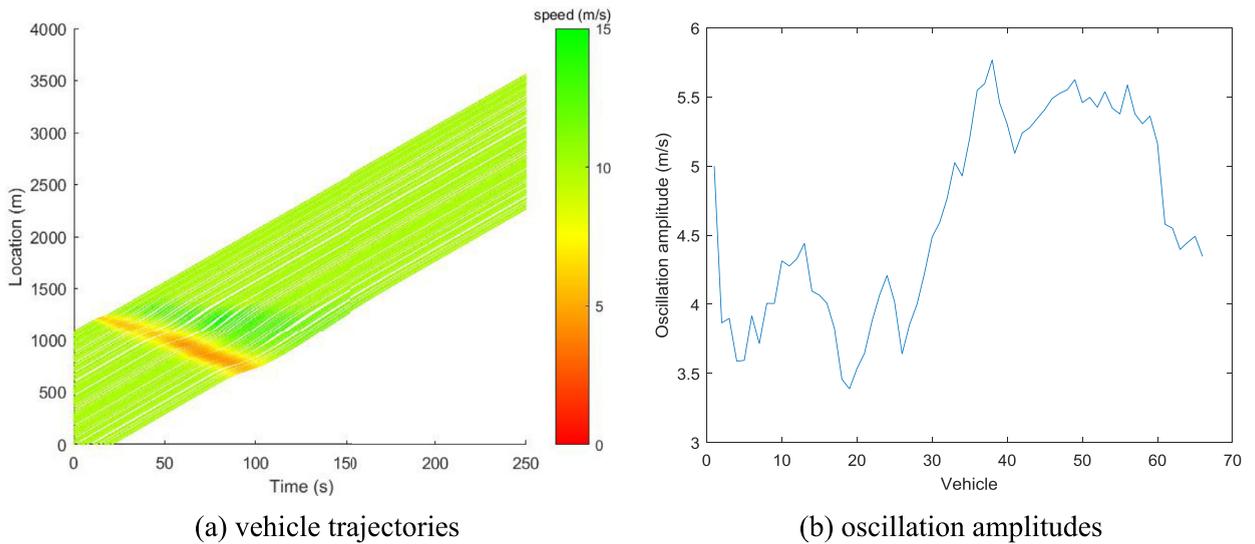


Fig. 11. Reproduced oscillation with heterogeneous platoon.

We then examine the oscillation characteristics using the calibrated parameter sets for IDM. Two types of platoon setup are considered according to the calibration methods. The first setup assigns vehicles to move with different calibrated parameter sets of IDM (in the order of the vehicles in the selected real platoon); in other words, a heterogeneous platoon. The second setup is a platoon of identical vehicles travelling with the parameter set that is calibrated by the second calibration method; in other words, a homogenous platoon. Same with the previous setup, the first vehicle drives at equilibrium speed and experiences one disturbance (it first decelerates and then accelerates 1 m/s^2 for the same time duration), and then returns to the original equilibrium speed. The results of the simulation setups are then analysed for the oscillation amplitude and oscillation type (as defined in Section 3.1).

Fig. 11 shows the oscillation reproduced with the heterogeneous platoon, given the baseline scenario where the equilibrium speed is 10 m/s , the deceleration duration is 5 s , and the deceleration of disturbance is 1 m/s^2 (the oscillation amplitude and speed drop of the first vehicle is 5 m/s). Fig. 11(a) presents the vehicle trajectories with obvious slow-and-go wave, while Fig. 11(b) plots the oscillation amplitudes of each vehicle. As indicated by Fig. 11(b), the oscillation amplitudes of vehicles first maintain a relatively lower level (around 4 m/s), and then increase to a higher level (around 5.5 m/s), thus showing a growing oscillation.

Combining this data with that given in Fig. 9, we can infer that the growth of oscillation amplitude is largely caused by the continuous and intense instability of Vehicles 19–34. The largest oscillation amplitude and speed drop in the platoon is 5.77 m/s . According to the definitions of oscillation types, the reproduced oscillation belongs to Type IV, the most negative type. Despite the oscillation amplitudes, the empirical oscillation (depicted in Fig. 10) and the simulated scenarios have a similar growth trend. This demonstrates the feasibility of the first calibration method (i.e. one parameter set for one pair of vehicles) and the heterogeneous platoon simulation.

Table 3
Impact of deceleration duration on the heterogeneous platoon oscillation.

Deceleration duration (s)	Deceleration (m/s ²)	Oscillation amplitude of first vehicle (m/s)	Average oscillation amplitude (m/s)	Type
3	1	3	1.44	II
5	1	5	4.64	IV
8	1	8	9.31	IV
3	5/3	5	3.13	II
8	5/8	5	6.07	IV

As determined in Section 3.2.2, the deceleration of the disturbance has an obvious impact on the oscillation. Here, we also investigate the impact of different deceleration durations (i.e. 3, 5, and 8 s) on the reproduced oscillation, while other setups are consistent with the baseline scenario. For the sake of clarity, the results of average oscillation amplitude of the platoon, and the oscillation types for different scenarios are compared in Table 3. With the increase in deceleration duration, the oscillation changes from Type II to Type IV. Note that the oscillation amplitude of the first vehicle varies with the deceleration duration. However, by relating the oscillation amplitude of the first vehicle and the average oscillation amplitude, we can still recognize that greater deceleration duration results in a more negative type of oscillation. Moreover, to fully reveal the impact of oscillation duration, we have conducted additional numerical experiments while keeping the oscillation amplitude of the first vehicle unchanged. The results indicate that, with the same oscillation amplitude of the first vehicle, as the deceleration duration increases, the average oscillation amplitude of the following vehicles increases and the oscillation type changes from Type II to Type IV, namely a longer deceleration duration causes a more negative type of oscillation.

For a complete analysis, while it is impossible to compare the simulated oscillation characteristics with the observed ones (because the oscillations in NGSIM are naturally generated by heterogeneous platoons), we also undertake a similar analysis and present characteristics of oscillations that are simulated by a homogeneous platoon (i.e. where a platoon of identical vehicles travel with one calibrated parameter set). With the same baseline scenario, the oscillation reproduced with the homogeneous platoon is shown in Fig. 12. A smoothly growing oscillation can be found in Fig. 12(a), while Fig. 12(b) indicates that the oscillation amplitude first drops and then rises along the platoon. This is mostly because of the used temporary perturbation. In real traffic a temporary perturbation can be approximated using a composition of many harmonic contributions of which most are in short-wavelength modes and stable. Short-wavelength (stable) modes will prevail in the beginning and long-wavelength (low frequency but unstable) modes will dominate after 50 or even 200 vehicles. Therefore, the oscillation amplitudes initially shrink from vehicle to vehicle and then grow. And this also leads to the fact that we are able to always observe stable platoon with a small size even when using unstable parameters. This highlights the difference between infinitely-long linear instability and finite-platoon oscillations which is one motivation of this study.

As the largest oscillation amplitude and speed drop in the platoon is 4.97 m/s (which is constrained by the oscillation amplitude and speed drop of the first vehicle), the reproduced oscillation goes to a Type II. Furthermore, the reproduced oscillation type is consistent with the theoretical results of Eqs. (7) and (17).

The impact of different deceleration durations on the reproduced oscillation is also studied for the second simulation scenarios, and the results are presented in Table 4. The same conclusion can be drawn from these scenarios. A comparison with

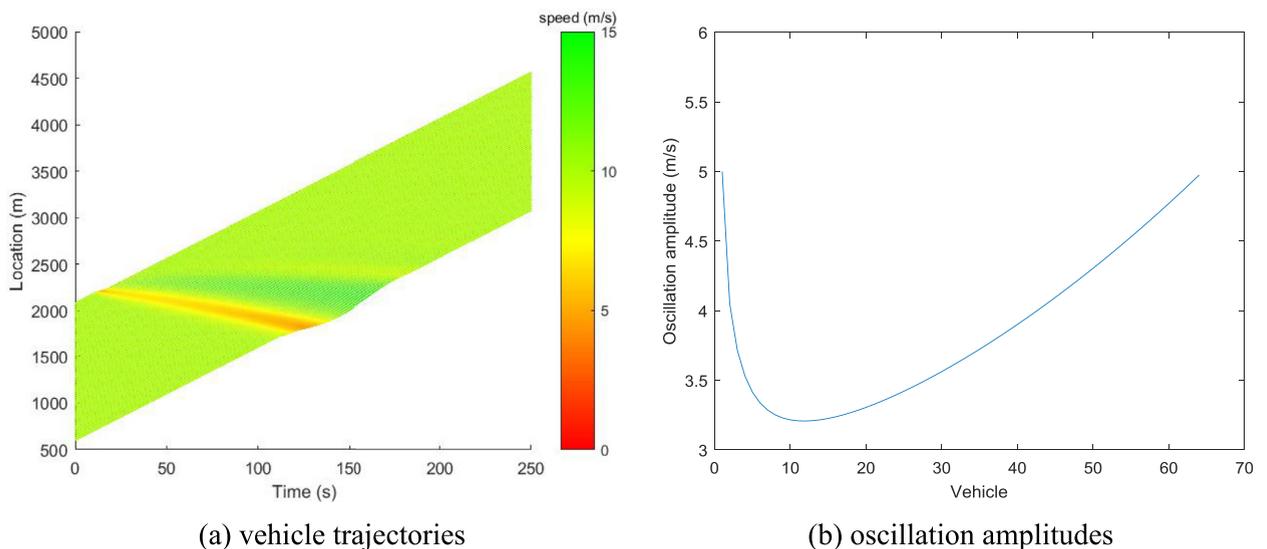


Fig. 12. Reproduced oscillation with homogenous platoon.

Table 4
Impact of deceleration duration on the homogenous platoon oscillation.

Deceleration duration (s)	Deceleration (m/s ²)	Oscillation amplitude of first vehicle (m/s)	Average oscillation amplitude (m/s)	Type
3	1	3	1.24	II
5	1	5	3.87	II
8	1	8	8.95	IV
3	5/3	5	2.44	II
8	5/8	5	5.36	IV

the results of the empirical oscillation in the heterogeneous platoon indicates that the homogeneous platoon downgrades average oscillation amplitudes, and (somehow) smooths the traffic flow as the individualities within the whole platoon are compressed.

4.4. Smoothing traffic oscillations by improving stability profiles of individual vehicles

With the calibrated IDM, we now demonstrate how the evolution of a traffic oscillation can be altered by changing individual vehicles’ stability profiles in a connected environment.

4.4.1. The case of the heterogeneous platoon

First, the heterogeneous platoon setup depicted in the previous section is used. For this platoon, the most unstable vehicles are selected according to their theoretical stability values, and then changed to more stable vehicles by using different parameter values in a connected environment. Since adjustable parameters can be different for different vehicle types (more discussion is presented later), two types of vehicles are considered: connected human-driven vehicles and connected and automated vehicles.

Generally, connected vehicles can communicate with neighbouring vehicles (V2V) and infrastructure (V2I). Detailed information (e.g. spacing, speed, acceleration of previous vehicles) or advisory information (accelerating or braking hard) might be provided to the subject vehicle, and the driving behaviour of the subject vehicle could change significantly, i.e. desired time headway is likely to increase (Sharma et al., 2019c). However, the principle of CF and the fundamental structure of modelling largely remain unchanged. In this study, we considered both the connected human-driven vehicles and connected and automated vehicles. We did not assume what kind of information would be provided to drivers but rather focused on what would be the most likely behavioural consequences (as a result of the connected environment) that need to be considered in a CF model, in other words, what are the variables of a CF model should be modified/re-calibrated. As per Kesting et al. (2008), although human-driven vehicles and automated vehicles have different features (e.g. reaction time, estimation errors, and multi-anticipation), it is suggested that they share similar driving behaviour. Therefore, we use basic IDM as the CF model for both types of vehicles. For the basic IDM without the acceleration exponent, five parameters can be optimized theoretically: desired speed v_0 ; the minimum gap at the standstill situation s_0 ; the desired time gap T ; the maximum acceleration α ; and the comfortable deceleration β . In order to effectively influence individual vehicles’ driving behaviour for the purpose of improving their stability and consequently smoothing traffic oscillations, we need to carefully select one of these five parameters for consideration as a control factor. To determine this selection, the sensitivities of all parameters on stability of IDM are first evaluated according to Eqs. (12)–(14), where the default parameter values are: $v_0=120$ km/h (i.e. 33.3 m/s); $s_0=2$ m; $T=1$ s; $\alpha=1$ m/s²; $\beta=1.5$ m/s²; $v_e=10$ m/s. The results are shown in Fig. 13.

As shown in Fig. 13, with the exception of the desired speed, the varying trends of stability over most parameters are monotonic (i.e. increasing or decreasing). With the increase in desired speed, the stability of IDM first increases, and then decreases. Although drivers’ desired speed is generally much higher than the value with the maximum stability (12 m/s in this scenario), thus it seems that the non-monotonic trend of desired speed is trivial, it is important to investigate the impact trend of desired speed on the stability of a platoon in order to find out the optimal value of desired speed for easing traffic congestion, especially in a connected environment. Because in the connected environment, to improve the stability of a single vehicle and ultimately smooth out a platoon’s oscillation, one efficient measurement is to change the desired speed as demonstrated in Tables 5 and 6. A larger desired time gap and maximum acceleration, or a smaller minimum gap and comfortable deceleration, lead to a more stable CF model. It is worth noting that with the default parameter values – which are typical values for human drivers on a highway (Treiber and Kesting, 2013b) – the stability of IDM is -1.2911 . This falls into the unstable region. However, as the stability of the default parameter set is at the average stability level of the calibrated parameter sets, it is still used as the initial parameter set of connected/automated vehicles in the following analysis.

Meanwhile, when connected and automated vehicles are used to smooth traffic oscillations, factors that can be influenced by connectivity and/or automation at the individual level vary depending on whether a vehicle is operated by a human or not. Thus, in the case of the heterogeneous platoon, we consider two scenarios: the use of connected (but human-driven) vehicles to smooth traffic, and the use of automated vehicles for the same purpose.

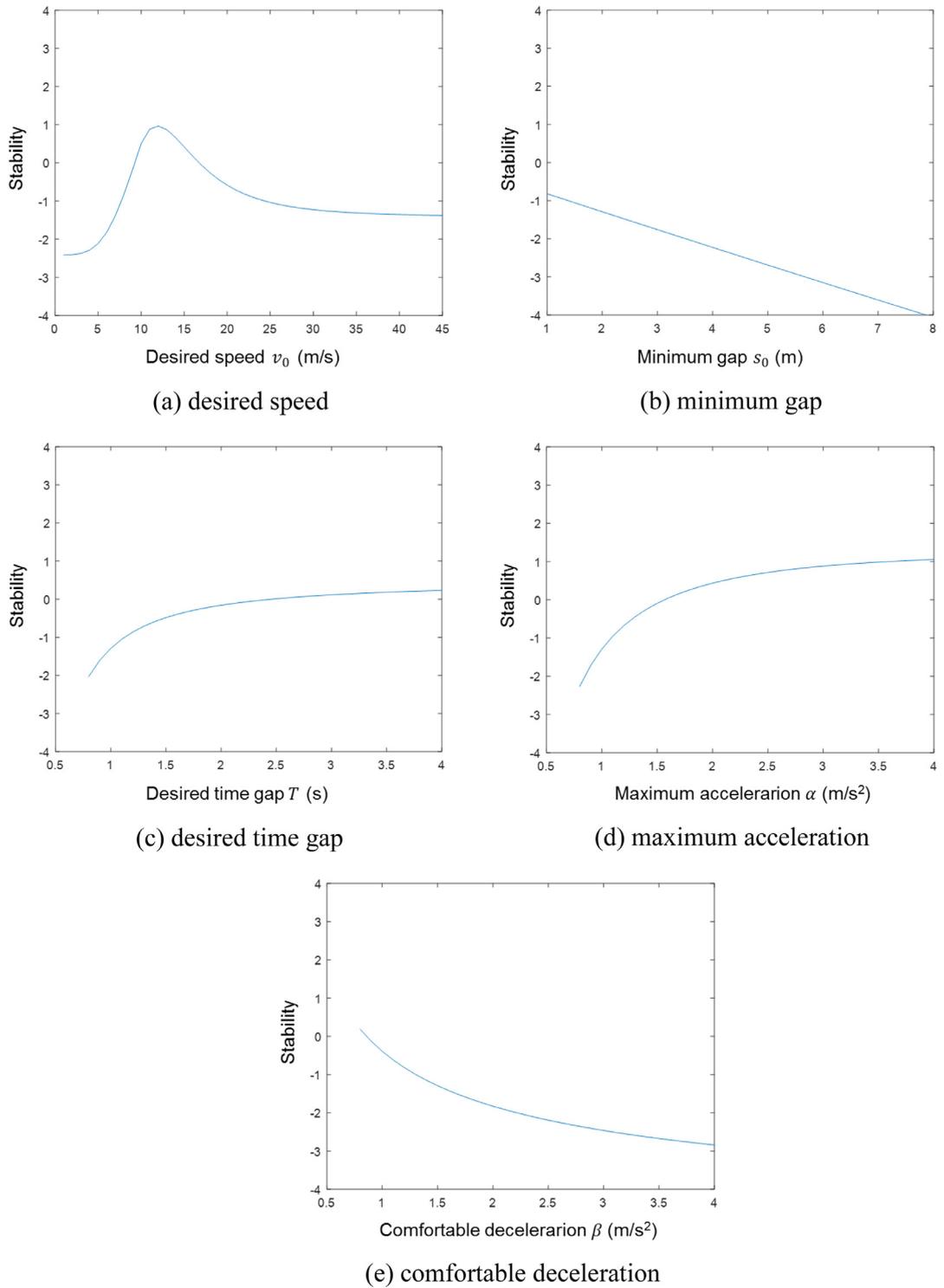


Fig. 13. The sensitivities of IDM parameters on stability.

Table 5
Impact of IDM's parameters on heterogeneous platoon oscillation.

Selected vehicles	Desired time gap (s)	Average oscillation amplitude (m/s)/ Type	Maximum acceleration (m/s ²)	Average oscillation amplitude (m/s)/ Type	Desired speed (m/s)	Average oscillation amplitude (m/s)/ Type
	Connected human-driven vehicle/Automated vehicle		Automated vehicle		Automated vehicle	
<−20 (6 vehicles)	0.8	4.64 (IV)	0.8	3.72 (III)	45	3.99 (III)
	1	3.68 (III)	1	3.60 (II)	35	3.24 (III)
	1.5	3.37 (II)	1.5	3.42 (II)	25	3.92 (III)
	2.0	3.11 (II)	2.0	3.32 (II)	15	3.36 (II)
<−10 (9 vehicles)	0.8	3.48 (II)	0.8	3.51 (III)	45	3.62 (III)
	1	3.35 (II)	1	3.35 (II)	35	3.10 (III)
	1.5	2.99 (II)	1.5	3.10 (II)	25	3.65 (III)
	2.0	2.64 (II)	2.0	2.96 (II)	15	2.96 (II)
<−5 (12 vehicles)	0.8	3.17 (II)	0.8	3.19 (II)	45	3.17 (III)
	1	2.96 (II)	1	2.96 (II)	35	2.82 (III)
	1.5	2.46 (II)	1.5	2.60 (II)	25	3.24 (II)
	2.0	2.06 (II)	2.0	2.41 (II)	15	2.37 (II)

Table 6
The impact of IDM parameters on homogeneous platoon oscillation.

Desired time gap (s)	Average oscillation amplitude (m/s)/ Type	Maximum acceleration (m/s ²)	Average oscillation amplitude (m/s)/ Type	Desired speed (m/s)	Average oscillation amplitude (m/s)/ Type
0.8	5.96 (IV)	0.8	6.31 (IV)	45	5.03 (IV)
1	4.83 (IV)	1	4.83 (IV)	35	4.88 (IV)
1.5	2.72 (II)	1.5	2.83 (I)	25	4.31 (III)
2.0	1.81 (I)	2.0	2.20 (I)	15	2.24 (I)

(1) Using connected vehicles to smooth traffic

Sharma et al. (2019c) compared the microscopic traffic flow parameters of connected human-driven vehicles with those of traditional vehicles (without connectivity), and reported that time headway increases in the connected environment. In addition, because time gap and headway are directly proportional to each other, Sharma et al. (2019b) modelled the impact of connectivity by modifying the desired time gap. Additionally, the other four parameters in IDM are usually hard to adjust for human drivers. Thus, we only consider the desired time gap as a controllable factor for connected human-driven vehicles. Four representative values of the desired time gap are employed here: 0.8 s, 1 s, 1.5 s, and 2 s (Kesting et al., 2008; Treiber and Kesting, 2013b).

Three groups of unstable vehicles are selected according to their degree of stability: (1) stability < −20 (6 vehicles); (2) stability < −10 (9 vehicles); (3) stability < −5 (12 vehicles). By replacing these unstable vehicles with the default parameter set (since its stability is at the average level of the calibrated parameter sets) and varying the desired time gap (to further change its stability), the oscillation characteristics and oscillation types are evaluated. The results are shown in Table 5. Compared with the original oscillation produced using IDM (the average oscillation amplitude is 4.64 m/s /Type IV), we can see that the oscillation's severity is largely mitigated. This is because the average oscillation amplitude becomes smaller when the stability of some individual vehicles increases. The more (unstable) vehicles are stabilized, the smoother the oscillation becomes. Moreover, a larger desired time gap also results in a smoother oscillation; this is consistent with its impact on stability. This result is also consistent with Monteil et al. (2018) conclusion.

(2) Using automated vehicles to smooth traffic

For automated vehicles, all five parameters can be easily optimized. However, the minimum gap and comfortable deceleration are not considered in this study. For the minimum gap, although Fig. 13(b) indicates that a smaller minimum gap leads to a higher stability, the default value (2 m) is already quite small for a safe driving. For comfortable deceleration, although a smaller comfortable deceleration leads to better stability and comfort, it is suggested that very small values of comfortable deceleration (less than about 1 m/s²) are not meaningful, since driving behaviour becomes more defensive when approaching a standing obstacle (Treiber and Kesting, 2013b). The default value (1.5 m/s²) is thus appropriate. Therefore, for automated vehicles, we consider that the desired speed, maximum acceleration, and desired time gap are controllable. These results are also presented in Table 5.

As shown in Table 5, replacing the unstable vehicles reduces the oscillation's severity. Similar to the impact of desired time gap (discussed above), the maximum acceleration also has a positive impact on the oscillation. When the desired speed decreases, although the oscillation type becomes less severe, the average oscillation amplitude does not necessarily become smaller.

In addition, although the current categorisation of four oscillation types is derived from the homogenous platoon assumption, three of the four oscillation types (Type II to IV) can be reproduced with the heterogeneous platoon by changing

some parameters of some vehicles, as indicated by [Table 5](#). The four measures of oscillation proposed remain distinguishable in the more general setting. The fact that Type I oscillation (similar to the stable condition) is difficult to be replicated using the heterogeneous platoon setup implicitly underscores the necessity of classifying oscillation into these four types.

4.4.2. The case of the homogeneous platoon

Similar strategies are implemented when simulating the homogeneous platoon oscillation. For the homogeneous platoon, vehicles are connected and automated. We replace the original calibrated parameter set with the default parameter set, change the values of three controllable parameters (i.e. desired time gap, maximum acceleration, and desired speed), and assess the impact on the oscillation. Results are presented in [Table 6](#).

The results in [Table 6](#) accord with the impact of IDM's parameters on stability. The oscillation is alleviated when vehicles' stability increases. With the exception of the impact of desired speed, similar conclusions can be drawn from the heterogeneous platoon analysis. In the homogeneous platoon, a small desired speed always leads to less severity of oscillation.

In summary, our analysis shows that desired time gap and maximum acceleration are two promising parameters that can be used in a connected and/or automated environment to significantly smooth traffic oscillations. Adjusting desired time gap, in particular, can be an effective strategy for improving CF stability and smoothing traffic oscillations. However, a significant increase in desired time gap could hamper road capacity. Therefore, a balance between stability and efficiency of traffic flow needs to be carefully considered in applications.

5. Conclusions and discussions

This paper links two fundamental concepts in traffic flow modelling – CF stability and traffic oscillation – with the ultimate goal of utilizing both CF instability analysis and oscillation analysis complementarily for the purpose of compressing and easing traffic congestion.

First, the relationship between CF instability and traffic oscillation was rigorously examined and established. In this process, some of the assumptions (e.g. platoon length and long-wavelength) usually adopted in stability analysis were relaxed. Oscillation was categorized into four types: amplitude-decay oscillation, amplitude-ceiling oscillation, speed-deviation ceiling oscillation, and speed-deviation growth oscillation. Each type was compared with CF instability. Results show that the number of vehicles in platoon, and deceleration duration of disturbance significantly impact the differences between CF instability and different types of oscillation with a logarithm relationship. The stability criteria were then modified by incorporating these two factors to form a set of oscillation criteria for identifying oscillation types. Our analysis shows that the oscillation criteria can distinguish different types of oscillation with high accuracy.

Such knowledge enabled us to utilize both CF instability analysis and oscillation analysis complementarily for the purpose of compressing and easing traffic congestion. To demonstrate this, a platoon of vehicles that experienced an oscillation in the NGSIM data was used as a case study. First, IDM was calibrated using trajectories of these vehicles. Two calibration approaches were used: one parameter set for one vehicle, and one parameter set for the platoon. We then generated the oscillation with two platoon setups: a heterogeneous platoon and a homogeneous platoon. In the case of the heterogeneous platoon, we considered two scenarios: the use of connected (but human-driven) vehicles to smooth traffic, and the use of automated vehicles for the same purpose.

Through these analyses, we demonstrated how a traffic oscillation's evolution can be altered by changing individual vehicles' stability profiles in a connected environment. More specifically, desired time gap, maximum acceleration, and desired speed were thoroughly studied to determine their impact on stability and, consequently, their impact on traffic oscillations in a connected and/or automated environment. Results show that higher stability of some individual vehicles can alleviate the oscillation severity for the platoon. Desired time gap and maximum acceleration are two promising parameters that can be used to improve individual vehicle stability, and to significantly smooth the oscillation of the platoon.

Of all the factors considered in our analysis, the adjustment of desired time gap is the most effective means of smoothing traffic oscillations. However, at the same time, increasing the desired time gap can have a negative impact on efficient traffic operation. Thus, a balance between stability and efficiency of traffic flow needs to be carefully considered when designing optimization and control strategies in practice. This is an important topic for future research.

Regarding the applicability of oscillation criteria developed in this paper, as they are calibrated based on the simulation results of homogenous platoon, the oscillation criteria cannot be directly applied in a heterogeneous platoon. However, the relationship between traffic oscillation and CF string instability can be used in real traffic when developing new control strategies with connected and automated vehicles by utilising the impact mechanism of platoon size and disturbance duration on oscillation severity. We could assign specific control parameters to vehicle according to the control objective rather than make the vehicle strictly string stable, as we can achieve less severe oscillation objective with lower stable parameters, which means less adjustment of driving behaviours.

Driver's reaction time is an important cause of CF instability and traffic oscillations, as demonstrated in [Sun et al. \(2018\)](#). However, regarding the stability analysis for CF model with reaction time (i.e. time-delayed CF model in [Sun et al., 2018](#)), the stability criteria derived from different methods are often inconsistent. Also, the main objective of this study is to demonstrate how to link the stability analysis and oscillation analysis by modifying the well-established stability criterion. Therefore, reaction time was not incorporated into IDM in this study. One can easily extend IDM into multi-anticipative IDM or CAV-IDM that incorporates multiple preceding vehicles (within the communication range) in the model. Since it was

demonstrated that the connectivity can improve vehicles' stability (Sun et al., 2018) and that increasing the stability of a single vehicle can smooth the oscillation of the platoon (this study), the positive influence of multi-anticipative IDM or CAV-IDM on smoothing out oscillations can be expected.

Moreover, traffic oscillations are usually considered as the exhibition of linear string instability as disturbances experienced by road users are often small. The linear string stability is thus the focus in this study. However, for large perturbations that can also occur in real traffic (e.g. inconsiderate hard braking or lane changes), the linear string stability analysis is not suitable anymore. Several studies in the literature have implemented nonlinear stability analysis in traffic flow (Ge et al., 2005; Monteil et al., 2014; Orosz et al., 2010a). However, the conditions for deriving equations for shock waves (e.g., Korteweg-de-Vries (KdV) or modified KdV equation) are very restrictive and are nearly never satisfied in real traffic situations, and numerical simulation is an alternative approach for nonlinear analysis that needs further investigation (Sun et al., 2018; Treiber and Kesting, 2013b).

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Appendix

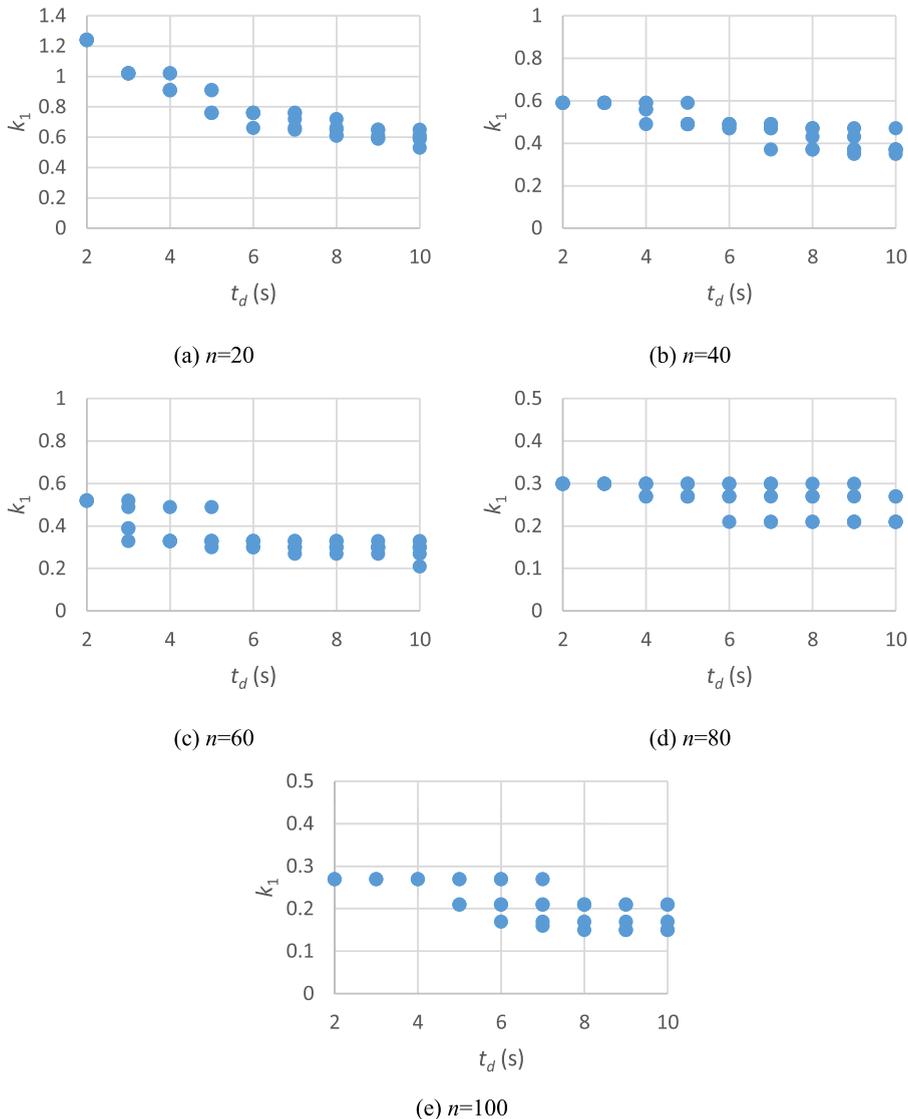


Fig. A1. The relationship between k_1 and t_d for different numbers of vehicles.

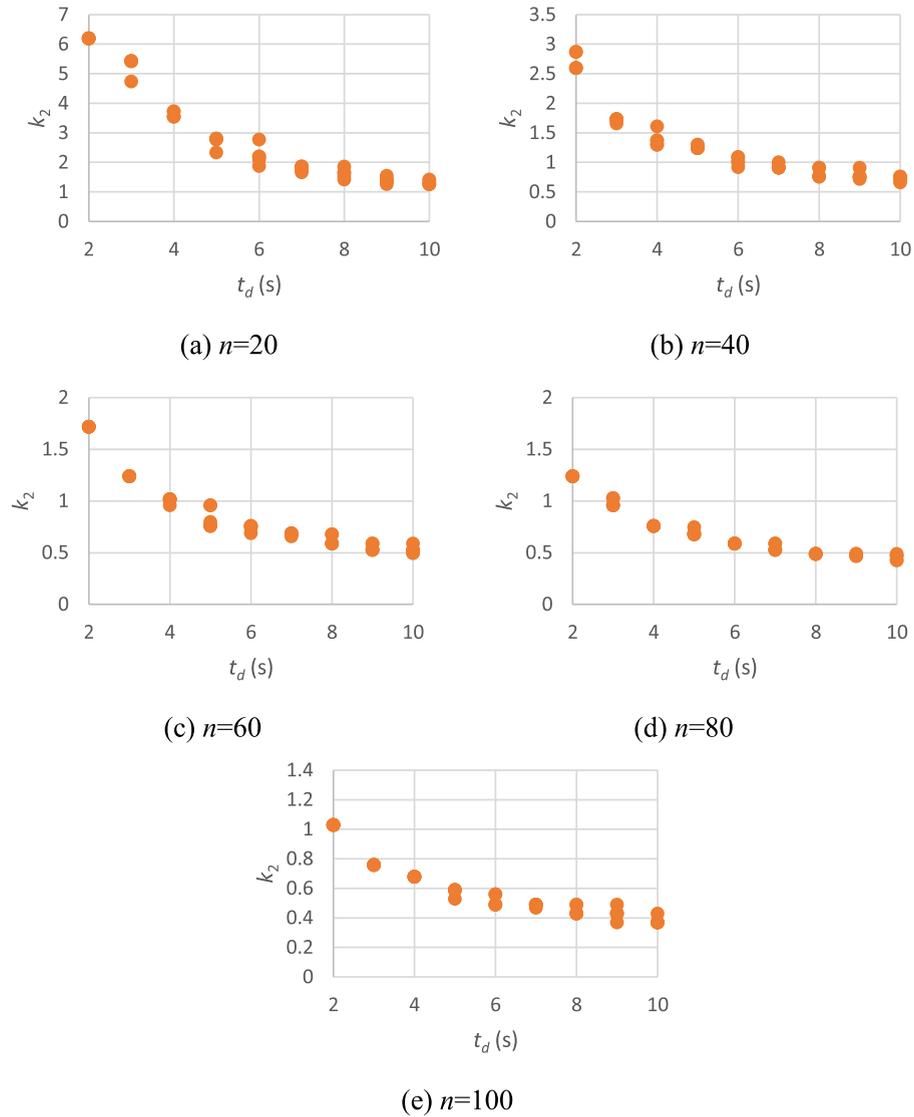


Fig. A2. The relationship between k_2 and t_d for different numbers of vehicles.

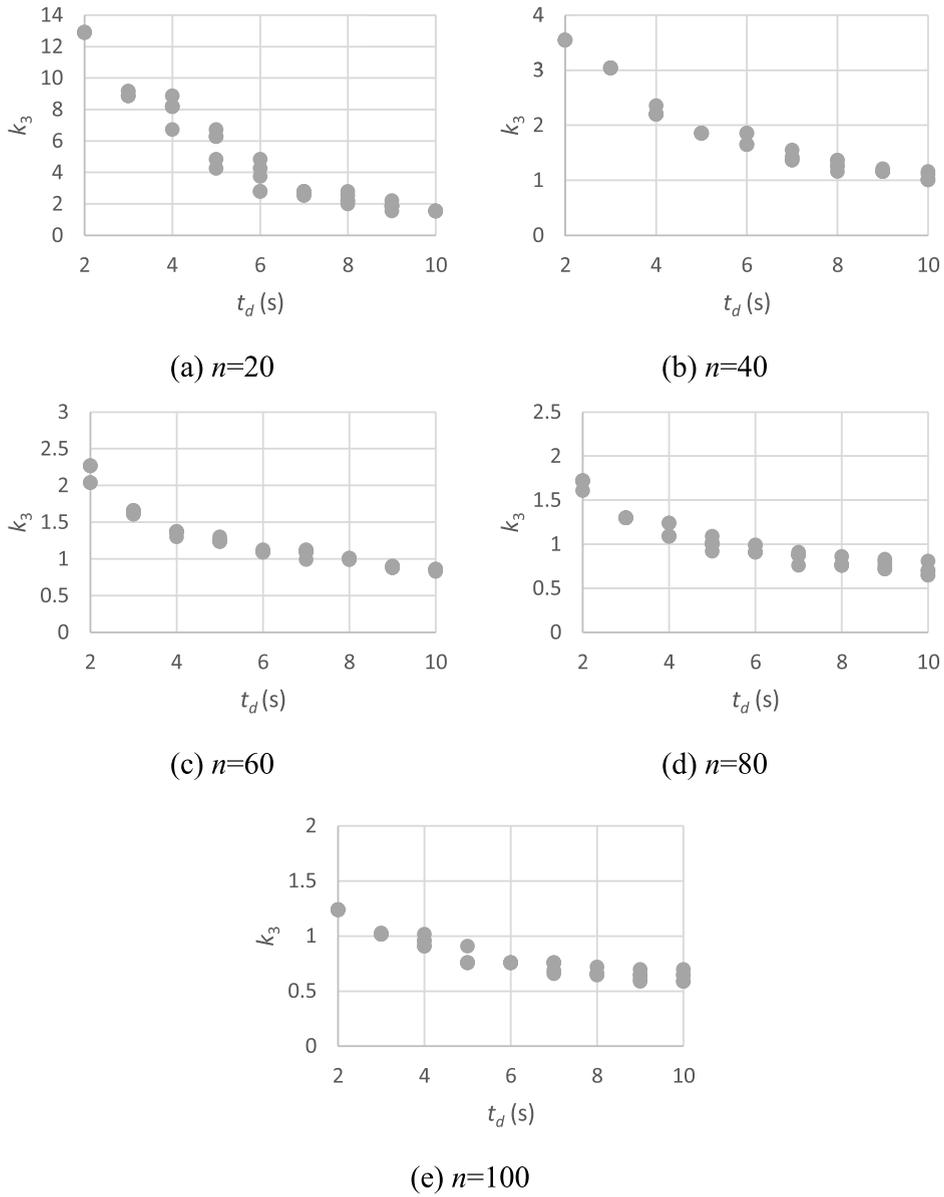


Fig. A3. The relationship between k_3 and t_d for different numbers of vehicles.

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