Incorporating human-factors in car-following models: A review of recent developments and research needs

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Abstract: Over the past decades there has been a considerable development in the modeling of car-following (CF) behavior as a result of research undertaken by both traffic engineers and traffic psychologists. While traffic engineers seek to understand the behavior of a traffic stream, traffic psychologists seek to describe the human abilities and errors involved in the driving process. This paper provides a comprehensive review of these two research streams.

It is necessary to consider human-factors in CF modeling for a more realistic representation of CF behavior in complex driving situations (for example, in traffic breakdowns, crash-prone situations, and adverse weather conditions) to improve traffic safety and to better understand widely-reported puzzling traffic flow phenomena, such as capacity drop, stop-and-go oscillations, and traffic hysteresis. While there are some excellent reviews of CF models available in the literature, none of these specifically focuses on the human factors in these models.

This paper addresses this gap by reviewing the available literature with a specific focus on the latest advances in car-following models from both the engineering and human behavior points of view. In so doing, it analyses the benefits and limitations of various models and highlights future research needs in the area.

Keywords: car-following; driver behavior; human factors; risk taking; driver error

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1. INTRODUCTION

Car-following (CF) rules describe longitudinal interactions of vehicles on the road. The CF concept was first introduced by Pipes and Reuschel (Pipes, 1953; Reuschel, 1950). It can be defined as 'the decision of the driver to follow the preceding vehicle efficiently and safely'. Over the past decades, traffic engineers and traffic psychologists have contributed to the development of CF behavior modeling. Traffic engineers seek to understand characteristics of a traffic stream and apply Newtonian laws of motion to approximate CF behaviors in what this paper refers to (for the convenience of discussion) as 'Engineering CF models'. Traffic psychologists, on the other hand, are motivated to describe the human abilities and errors involved in CF, and their impact on traffic safety. Another mainstream driver behavior – lane-changing maneuvers – is reviewed in Zheng (2014) and is beyond the scope of this paper.

A large number of Engineering CF models have been developed in an attempt to describe CF behavior under a wide range of traffic conditions, ranging from free-flow to extreme situations. Some of these models have been used in commercial packages of microscopic traffic simulations (Barceló, 2010), and to guide the design of advanced vehicle control and safety systems (Yang and Peng, 2010). However, the limitations of Engineering CF models were the subject of spirited debate after the publication of Brackstone and McDonald's (1999) historical review of car-following models. In a commentary of this review, Hancock (1999) criticized the fact that the psychologically plausible characterization of how humans think about, and solve, the driving problem is not observed in these CF models.

Each driver is different so as their driving styles and risk-taking capabilities. Age and gender, for example, play an important role in the perception of risky driving situations. In addition, particular driving needs can influence aggressive driving, which is a potential source of driving error. While research shows that driver error contributes to up to 75% of all roadway crashes (Stanton and Salmon, 2009), few CF models can capture driver behavior in various driving conditions, especially in crash-prone conditions, such as traffic breakdowns, the undertaking of risk-taking behaviors, distraction, and adverse weather conditions.

To address this serious issue, a richer representation of the cognitive processes engaged during CF is required to describe driver responses, and the consequences of these responses, in adverse driving conditions. Moreover, CF models with the capability of mimicking a driver's mistakes and, consequently, with the ability to generate crash or near-crash scenarios can be important tools for evaluating safety-related technologies and policies. Unfortunately, most Engineering CF models do not include such scenarios.

Given the importance of the human factor in the driving process, it is necessary to integrate the latest CF modeling advances from both engineering and psychological perspectives, and to bridge any gaps or inconsistences in these perspectives. Such a union will be of great value in transportation research, especially in micro-simulation models for better prediction of driving behavior. This paper explores the existing CF models and their advances in describing human driving behavior.

Although some excellent reviews of CF models are available (Brackstone and McDonald, 1999; Hamdar, 2012; Olstam and Tapani, 2004; Panwai and Dia, 2005; Toledo, 2007), all have their limitations. For example, Brackstone and McDonald (1999) review CF models developed before 1999. Since then, however, there have been notable advancements in CF modeling. Furthermore, the Brackstone and McDonald review (1999) ignores cellular automation (CA)-based CF models, and their review is limited to Engineering CF models only. Similar conclusions can be drawn from the reviews by Olstam and Tapani (2004), Panwai and Dia (2005), and Toledo (2007). (Note, however, that Toledo (2007) does include CA-based CF models). In contrast, few efforts are observed on identifying human factors responsible for car-following with two exceptions. Hamdar (2012) summarized a list of human factors and situational environmental factors which may affect CF behavior. In a recent review, Treiber and Kesting (2013) described seven human factors (finite reaction time, estimation error, imperfect driving, spatial and temporal anticipation, context sensitivity and perceptual threshold) which could affect CF behavior, and applied them to a CF model using some hypothetical cases.

This paper provides a comprehensive review of the important recent developments in CF modelling from both engineering and human behavior perspectives. In particular, the paper focuses on notable efforts to integrate human behaviors into the traditional CF models, and on the future research that is needed to build on these efforts. For the sake of clarity and focus, the paper concentrates on representative CF models in the literature, rather than attempting to exhaustively cover all existing models.

To this end, the paper is organized as follows: Section 2 reviews notable traditional CF models and their extensions; Section 3 presents Engineering CF models that attempt to incorporate one or more human factors; and Section 4 discusses the major issues arising from these previous modeling attempts, determines what future research is needed in the area, and summarizes the conclusions arising from the review.

2. CAR-FOLLOWING MODELS: THE ENGINEERING PERSPECTIVE

Numerous mathematical models have been developed to describe CF behavior under a wide range of conditions. In general, these models are based on the stimulus-response framework that was first developed at the General Motors research laboratories (Chandler et al., 1958; Gazis et al., 1961). The framework assumes that each driver responds to a given stimulus according to the following relationship:

$$response = sensitivity \times stimulus$$

Over the years, various researchers have used different factors as the stimuli to explain the response (acceleration) of the subject vehicle. While varying notations are used in the literature, for the sake of consistency and clarity, the same notations are used throughout this paper (These are listed in Appendix).

2.1. GHR model and its extensions

Gazis-Herman-Rothery (GHR) CF models is probably the most studied models in the area of CF modeling. The first version is the linear CF model developed by Chandler et al. (1958) and Herman et al. (1959), as shown in Equation (1)

$$a_n(t) = \lambda . \Delta V_n(t - \tau_n) \tag{1}$$

where $a_n(t)$ is the acceleration of the subject vehicle n at time t, $\Delta V_n(t - \tau_n)$ is the speed difference between the subject vehicle and the preceding vehicle at time $(t - \tau_n)$, τ_n denotes the reaction time, and λ is a sensitivity parameter. The sensitivity parameter λ can have several functional forms

(a)	$\lambda = C,$	a constant
(b)	$\lambda = \begin{cases} C_1, & \Delta X_n \le \Delta X_{critical} \\ C_2, & \Delta X_n > \Delta X_{critical} \end{cases}$	a step function
(c)	$\lambda = C/\Delta X_n\ ,$	reciprocal spacing
(d)	$\lambda = C \cdot V_n/\Delta X_n,$	used in Edie's model (Edie, 1961)
(e)	$\lambda = C/\Delta X_n^2\ ,$	yields Greenshield's (Greenshields et al., 1935) macroscopic flow-density relationship

where, ΔX_n is the spacing from the preceding vehicle, $\Delta X_{critical}$ is a threshold specified by the modeler, V_n is the speed of the subject vehicle, and C, C_1, C_2 are constant. Gazis et al. (1961) combine the last three (c, d, e) functional forms of λ in a general expression of sensitivity, and propose a non-linear CF model, as defined in Equation (2)

$$a_n(t) = \alpha V_n(t)^{\beta} \frac{\Delta V_n(t-\tau_n)}{\Delta X_n(t-\tau_n)^{\gamma}} \tag{2}$$

where α, β, γ are parameters.

GHR models have been extensively studied (For a detailed review, see Brackstone and McDonald, 1999). The main advantage of GHR model is its simplicity. However, it was built upon several strong assumptions, and this leads to the serious limitations as being frequently reported by researchers (Siuhi and Kaseko, 2010). For example, identical reaction time for all drivers does not capture inter-driver heterogeneity; the human ability to perceive small changes in driving conditions, such as spacing and relative velocity, is overestimated; and single value estimation for each of the model parameters does not consider behavioral differences in different circumstances (such as acceleration or deceleration). In an attempt to overcome these limitations, several enhanced versions of the GHR model have been developed, as elaborated below.

Memory functions: Assuming that a driver reacts to the relative speed of the preceding vehicle over a period of time, rather than in an instant, Lee (1966) introduces a memory function into the linear GHR model to store the information of relative speed during CF, as shown in Equation (3)

$$a_n(t) = \int_0^t M(t-s)\Delta V_n(s)ds \tag{3}$$

where *M* represents a memory function; that is, the way a driver acts on information that has been collected over the driving period. This function is similar to a weighting function. Lee (1966) proposes several forms of the memory function, and analyzes the stability of the resulting response to periodic changes in the preceding vehicle's speed. Although the model removes unrealistic peaks in acceleration profile, the implementation of the model in traffic simulation is considerably more complex due to the need of maintaining an array of past conditions for each vehicle.

Acceleration and deceleration asymmetry: Herman and Rothery (1965) were the first to hypothesize that most passenger cars have a greater deceleration than acceleration capacity.

This was later confirmed by Subramanian (1996) and Siuhi and Kaseko (2010). In congested traffic, drivers are more sensitive to deceleration than to acceleration. Ahmed (1999) extends the GHR model to accommodate this acceleration/deceleration asymmetry. In this model, driver heterogeneity in terms of reaction time is also considered. In addition, two states of driving – free flow and CF – are modeled separately within the model. The state of driver behavior (that is, free-flow or car-following) is determined by comparing the headway (h_n) to a critical value (h_n^*) which is distributed among the drivers. If $h_n(t-\tau_n) \leq h_n^*$ then the vehicle is in the CF state; otherwise, it is in the free-flow state. The model is shown in Equation (4)

$$
\begin{aligned}
a_n^{cf,g}(t) &= \alpha^g \frac{V_n(t-\varphi\tau_n)^{\beta^g}}{\Delta X_n(t-\varphi\tau_n)^{\gamma^g}} k_n(t-\varphi\tau_n)^{\delta^g} \Delta V_n(t-\varphi\tau_n)^{\rho^g} + \varepsilon_n^{cf,g}(t) \\
a_n^{ff,g}(t) &= \lambda^{ff}\left[\tilde{V}_n(t-\tau_n) - V_n(t-\tau_n) + \varepsilon_n^{ff}(t)\right]
\end{aligned} \tag{4}
$$

where *cf* and *ff* refer to CF and free-flow states respectively; g □ [acceleration, deceleration]; $k_n(t-\varphi\tau_n)$ is the traffic density ahead of the subject vehicle within its view (a visibility distance of 100m was used) at time $(t-\varphi\tau_n)$; $\varphi \in [0,1]$ is a sensitivity lag parameter; λ is the constant sensitivity; $\tilde{V}_n$ is the desired speed; and ε_n^{cf} and ε_n^{ff} are normally distributed error terms for CF and free-following states, respectively.

Koutsopoulos and Farah (2012) discovered some ambiguity in the previous assumption of the GHR model, where it is assumed that drivers accelerate when the speed difference relative to the preceding vehicle is positive, and decelerate when the speed difference is negative. In fact, after analyzing two existing traffic flow databases (Next Generation Simulation (Alexiadis et al., 2004), and Federal Highway Administration (FHWA, 1985)) they found that, in many cases, the opposite is true. Hence, they relax the assumption and extend the GHR model to consider three states of driving: accelerating, doing nothing, and decelerating.

Multiple-vehicle interaction: The models discussed above are based on the assumption that each driver reacts in some specific manner to some stimuli from the preceding vehicle. In the real world, however, drivers most likely adjust their behaviors according to their observations of more than one vehicle ahead. Multi-vehicle interaction was first introduced by Herman and Rothery (1965) and Bexelius (1968). Assuming that drivers follow more than one preceding vehicle, they extend the linear GHR model with added sensitivity terms for up to *m* vehicles ahead. The mathematical form of the model is presented in Equation (5)

$$
a_n(t) = \sum_{i=1}^{m} \alpha_i \Delta V_{n,n-1}(t-\tau_n) \tag{5}
$$

where $\Delta V_{n,n-1}(t-\tau_n)$ is the relative speed with respect to the nearest i^{th} leader at time $(t-\tau_n)$, and α_i is a parameter. Although the notion behind the model is a realistic one, this research direction received little attention in the literature until recently, when multi-vehicle interaction has re-gained some attention (Hoogendoorn and Ossen, 2005; Lenz et al., 1999; Peng and Sun, 2010; Treiber et al., 2006). (This is discussed later in this paper.)

Fuzzy-logic: Fuzzy-logic is applied to enhance the GHR model because its use is often reported to enable a better mimicking of the cognitive and perceptional uncertainties that drivers frequently encounter in real-world CF processes (Brackstone et al., 1998).

Aforementioned models assume that the drivers know their exact speed, their distance from other vehicles, and other situational factors. Clearly, this assumption is an unrealistic one. Fuzzy-logic-based models, on the other hand, acknowledge the imperfection of a driver's capability by dividing their perception into a number of overlapping fuzzy sets using predefined fuzzy-logics. For example, time headway of less than 0.5s is defined as *too close*. This definition can then be used in logical rules such as, *if too close, then use emergency deceleration*. Kikuchi and Chakroborty (1992) were the first to use this type of model to 'fuzzify' the traditional GHR model. More work with the fuzzy-logic-based model is reported in Wu et al. (2000). However, among many other issues, defining fuzzy sets and their associated membership functions is challenging (Ross, 2010), and makes the calibration and validation of fuzzy-logic-based CF models extremely difficult.

2.2. Desired measures models

Helly's model: According to the aforementioned CF models (Chandler et al., 1958; Gazis et al., 1961), for two vehicles that are travelling at the same speed, any value of spacing between them is acceptable. To address this shortcoming, Helly (1959) introduces a new assumption that each driver has a desired following distance, and the driver seeks to minimize both the speed difference and the difference between the actual space headway and the desired headway. The functional form of Helly's model is expressed in Equation (6)

$$\begin{aligned} a_n(t) &= \alpha_1 \Delta V_n(t-\tau_n) + \alpha_2\left[\Delta X_n(t-\tau_n) - \widetilde{\Delta X}_n(t)\right], \\ \widetilde{\Delta X}_n(t) &= \beta_1 + \beta_2 V_n(t-\tau_n) + \beta_3 a_n(t-\tau_n). \end{aligned} \tag{6}$$

where $\alpha_1, \alpha_1, \beta_1, \beta_2, \beta_3$ are parameters; $\widetilde{\Delta X}_n$ is the driver's desired following distance, which is assumed to be dependent on their speed and acceleration. However, Helly (1959) and other researchers (Koshi et al., 1992; Van Winsum, 1999; Xing, 1995) show that the desired following distance can be reasonably determined by using the speed of the subject vehicle alone (that is, $\beta_3 = 0$).

A non-linear extension of Helly's model in combination with the GHR model is proposed by Koshi et al. (1992) and, later, by Xing (1995). The general form of their model is presented in Equation (7)

$$\begin{aligned} a_n(t) = \alpha_1 \frac{\Delta V_n(t-\tau_1)}{\Delta X_n(t-\tau_1)^l} + \alpha_2 \frac{\left[\Delta X_n(t-\tau_2) - \widetilde{\Delta X}_n(t)\right]}{\Delta X_n(t-\tau_2)^m} - \gamma \sin\varphi \\ + \lambda[\tilde{V}_n - V_n(t-\tau_3)] \end{aligned} \tag{7}$$

where τ_1, τ_2, τ_3 are time lags, φ is the gradient difference in a sag, $\widetilde{\Delta X}_n$ is the desired following distance as a function of the vehicle speed, $\tilde{V}_n$ is desired speed, and $\alpha_1, \alpha_2, \gamma, \lambda, l, m$ are parameters. The first term of the model represents the standard driving situation, the second term describes acceleration from a standing queue, the third term controls the effect of gradient, and the fourth term represents acceleration in free-flow conditions. Note that, while the physical condition of the road in terms of gradient is considered in this model, horizontal curvature effect is neglected.

Intelligent driver model (IDM): One of the most popular models using desired measures is the intelligent driver model (IDM) proposed by Treiber et al. (2000). This model considers both the desired speed and the desired space headway, as defined in Equation (8)

$$a_n(t) = a_{\max}^{(n)}\left[1 - \left(\frac{V_n(t)}{\tilde{V}_n(t)}\right)^{\beta} - \left(\frac{\tilde{S}_n(t)}{S_n(t)}\right)^2\right] \quad (8)$$

where $a_{\max}^{(n)}$ is the maximum acceleration/deceleration of the subject vehicle n, $\tilde{V}_n$ is the desired speed, S_n is spacing between two vehicles measured from the front edge of the subject vehicle to the rear end of the preceding vehicle ($S_n = \Delta X_n - L_n$; where L_n is vehicle length), $\tilde{S}_n$ is the desired spacing, and β is a parameter. When preceding vehicle is far away, the third term in this equation becomes negligible small and the model performs as a free flow model where the desired speed of the driver governs the acceleration. Use of one equation ensures a smooth transition between free-flow and car-following situations. The desired space headway (or following distance) in IDM is dependent on several factors: speed, speed difference (ΔV_n), the maximum acceleration ($a_{\max}^{(n)}$), a comfortable deceleration ($a_{comf}^{(n)}$), the minimum spacing at the standstill situation ($S_{\text{jam}}^{(n)}, S_1^{(n)}$), and the desired time headway ($\tilde{T}_n$). Mathematically, the desired following distance can be calculated using Equation (9):

$$\tilde{S}_n(t) = S_{\text{jam}}^{(n)} + S_1^{(n)}\sqrt{\frac{V_n(t)}{\tilde{V}_n(t)}} + V_n(t)\,\tilde{T}_n(t) - \frac{V_n(t)\,\Delta V_n(t)}{2\sqrt{a_{\max}^{(n)} a_{comf}^{(n)}}} \quad (9)$$

The introduction of both a maximum acceleration and a comfortable deceleration rate prevents the model from producing unrealistically high accelerations/decelerations. This feature is absent in most of the earlier models. In calibrating this model, identical vehicles with the same acceleration and deceleration capability were used (a maximum value of 0.73m/s^2 was used). Reaction time is ignored in this model.

Later, Treiber and Helbing (2003) extended IDM to capture driver's adaptation effect to the surrounding environment using a memory function. Their model is called IDMM; that is, IDM with memory. The extension is based on the observation that, after being in congested traffic for some time, most drivers adapt their driving style; for example, by increasing their preferred time gap. Treiber and Helbing (2003) assume that the subjective level of service (λ_n) influences the desired time gap decision. Hence, the desired time gap $\tilde{T}_n(t)$ in Equation (9) is replaced by $T_n(\lambda)$. This is shown in Equation (10)

$$T_n(\lambda) = \tilde{T}_n[\beta_{\text{T}} + \lambda_n(1 - \beta_{\text{T}})];\ \beta_{\text{T}} = T_{\text{jam}}/\tilde{T}_n \quad (10)$$

where, β_{T} is an adaptation factor. For each driver, the subjective level of service (λ_n) is given by the exponential moving average of the instantaneous level of service experienced within the adaptation time (typically 600 sec).

The main difficulty of models with desired measures (for example, desired spacing, desired time headway, desired speed) is that most of the parameters are unobservable in nature, and this makes their estimation more challenging. Therefore, many of the models described in this sub-section were not empirically estimated using real traffic data.

2.3. Safety distance or collision avoidance models

Safety distance models differ from GHR models by hypothesizing that the driver reacts to spacing relative to the preceding vehicle, rather than to the relative speed. This idea was first proposed by Kometani and Sasaki (1959). In their model, the subject vehicle seeks to keep the minimum safety distance from the preceding vehicle, as shown in Equation (11)

$$\Delta X_n(t-\tau_n) = \alpha V_{n-1}^2(t-\tau_n) + \beta V_n^2(t) + \gamma V_n(t) + d \tag{11}$$

where, V_n and $V_{n-1}$are the speeds of the subject vehicle and the preceding vehicle, respectively; α, β, γ are parameters; and d is a constant which represents the minimum spacing and prevents the model from collisions. Later, Newell (1961) proposed a non-linear version of this model, which assumes that the speed of the subject vehicle is a non-linear function of the spacing to the preceding vehicles, as shown in Equation (12)

$$V_n(t) = V_{\max}[1 - \exp(-\lambda(\Delta X_n(t-\tau_n) + d)/V_{\max})] \tag{12}$$

where $V_{\max}$ and d are the maximum speed and the minimum space headway, respectively; λ is a parameter. Newell assumes different functional forms for acceleration and deceleration decisions. This model is directly dependent on density (spacing between vehicles), and this dependence might result in unrealistic accelerations or decelerations. To address this issue, Bando et al. (1995) modified Newell's model by controlling the change in speed. (This is discussed in Section 2.4 below.)

The most popular safety distance model was developed by Gipps (1981). The model assumes that the speed is selected by the driver in a way to ensure that the vehicle can be safely stopped in case the preceding vehicle should suddenly brake. Gipps' model includes two modes of driving: free-flow and CF. The driver chooses the smaller one from the speeds obtained from the free-flow and CF modes, as shown in Equation (13)

$$V_n(t+\tau_n) = \min\begin{cases} V_n(t) + 2.5\tilde{a}_n\tau_n\left(1 - V_n(t)/\tilde{V}_n\right)\left(0.025 + V_n(t)/\tilde{V}_n\right)^{1/2} \\ \tilde{b}_n\tau_n + \sqrt{\tilde{b}_n^2\,\tau_n^2 - \tilde{b}_n\left[2(\Delta X_n(t) - s_{n-1}) - V_n(t)\tau_n - \dfrac{V_{n-1}(t)^2}{\hat{b}}\right]} \end{cases} \tag{13}$$

where $\tilde{a}_n$is the desired acceleration, $\tilde{b}_n$ is the desired deceleration, s_{n-1} is the effective length of vehicle *n*-1 (length of the vehicle plus a safety distance into which the following vehicle is not willing to intrude even when at rest), $\hat{b}_n$is an estimate of the deceleration applied by the preceding vehicle (b_{n-1}), and $\tilde{V}_n$ is the desired speed of vehicle *n*. A constant reaction time τ_n is used for all vehicles. A smooth transition between free-flow and CF modes occurs most of the time, except when the leading vehicle brakes harder than anticipated (i.e. $b_{n-1} > \hat{b}_n$), when the preceding vehicle moves to an adjacent lane, or when a new vehicle moves in front of the subject vehicle from an adjacent lane. Besides its Newtonian equations of motion, Gipps' model offers some behavioral parameters, for example, the desired acceleration, desired deceleration and desired speed, reaction time, and estimation of the preceding vehicle's deceleration. It has been used in many simulation models, including AIMSUN (Barceló and Casas, 2005).

2.4. Optimal velocity model

The optimal velocity (OV) model, introduced by Bando et al. (1995) has received considerable attention in the CF literature. OV model assumes that each vehicle has an optimal (safe) velocity, which depends on the distance from the preceding vehicle, and that the acceleration of the n^{th} vehicle can be determined according to the difference between the actual velocity V_n, and the optimal velocity V_n^*. Mathematically, the model can be defined as in Equation (14)

$$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right] \tag{14}$$

where α is the constant sensitivity coefficient, and V_n^* is the optimal velocity and depends on the headway ΔX_n to the preceding vehicle, and can be defined as

$$V_n^*\left(\Delta X_n(t)\right) = V_0\left[\tanh\left(\frac{\Delta X_n(t) - L_{n-1}}{b} - C_1\right) + C_2\right]$$

where L_{n-1} is the length of the preceding vehicle (typically 5m), and b is the length scale while V_0, C_1 and C_2 are constant. Helbing and Tilch (1998) calibrated the OV model using the following optimal velocity function:

$$V_n^*\left(\Delta X_n(t)\right) = V_1 + V_2\tanh[C_1(\Delta X_n(t) - L_{n-1}) - C_2]$$

where V_1, V_2, C_1, C_2 are parameters, and their estimated optimal values are: V_1=6.75 m/s, V_2=7.91 m/s, C_1=0.13m^{-1}, C_2=1.57. Driver reaction time is not considered in the OV model described above, which has been updated in the later version (Bando et al., 1998), as shown in Equation (15):

$$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t-\tau_n)\right) - V_n(t-\tau_n)\right] \tag{15}$$

Although OV model was created to address the issue of the unrealistically high acceleration and deceleration observed in Newell's (1961) model, comparison with the field data shows that it still produces unrealistic accelerations and decelerations. The reason is that the optimal velocity is dependent on the following distance; hence, the density is still affecting the model. To handle unrealistic decelerations, Helbing and Tilch (1998) added velocity difference to the OV model; this comes into play when the velocity of the preceding vehicle is lower than that of the subject vehicle. They called the model the 'Generalized Force' (GF) Model, as presented in Equation (16)

$$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right] + \lambda\left(\Delta V_n(t)\right)\cdot H\left(-\Delta V_n(t)\right) \tag{16}$$

where H is a Heaviside function, whose value is 1 when the velocity of the preceding vehicle is lower than that of the subject vehicle, and 0 otherwise; and λ is the sensitivity constant. As both the acceleration and deceleration rate could be unreasonably high, Jiang et al. (2001) extended the GF model to consider both negative and positive velocity differences (that is, to explicitly consider velocity difference), and named it the 'Full Velocity Difference' (FVD) Model, as shown in Equation (17):

$$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right] + \lambda\left(\Delta V_n(t)\right) \tag{17}$$

Jiang et al. (2001) use the same OV function as is used in Helbing and Tilch (1998). However, the FVD model is indifferent to acceleration and deceleration behavior, which could be problematic. Previous research shows that drivers behave differently during acceleration and deceleration (as discussed in Section 2.1). Having a single parameter for both acceleration and deceleration might lead to an unrealistic situation where the subject vehicle brakes insufficiently, even if the distance to the preceding vehicle is extremely short. Thus, Gong et al. (2008) propose an asymmetric full velocity difference (AFVD) model by enabling different responses in acceleration and deceleration, as shown in Equation (18)

$$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right] + \lambda_1\left(\Delta V_n(t)\right) \cdot H\left(-\Delta V_n(t)\right) + \lambda_2\left(\Delta V_n(t)\right) \cdot H\left(\Delta V_n(t)\right) \tag{18}$$

where λ_1, λ_2 are sensitivity coefficients used for deceleration and acceleration respectively. Compared with the FVD model, the AFVD model takes longer time to become stable.

Davis (2003) simulated the OV model (Bando et al., 1998) using different reaction times. For a small reaction time 0.1s, flow was stable for a platoon of 100 vehicles. However, if the reaction time increased to 0.3s, only the first 14 vehicles avoided collision and the situation became worse for longer driver reaction times. This indicates that the OV model is unrealistically sensitive to delay time. To overcome this problem, the OV function for time-varying situations is modified by assuming that drivers can change the relative velocity as well as headway, as shown in Equation (19):

$$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t-\tau_n) + \tau_n \Delta V_n(t-\tau_n)\right) - V_n(t)\right] \tag{19}$$

For small reaction times, this model closely represents the original OV model. For long reaction times ($\tau_n \leq 1$s), the model performs well without any collisions for a platoon of 100 vehicles. The model calculates the relative distance and the relative velocity at time $(t-\tau_n)$, and calculates speed of the subject vehicle at time *t*, which is odd and needs a behavioral justification.

Lenz et al. (1999) extended the OV model by considering multi-vehicle interactions, as defined in Equation (20)

$$a_n(t) = \sum_{i=1}^{m} \alpha_i \left[V_n^*\left(\frac{\Delta X_{n,n-i}(t)}{i}\right) - V_n(t)\right] \tag{20}$$

where $\Delta X_{n,n-i}(t)$ is the spacing with respect to the nearest i^{th} leader at time *t*. For m=1, the above equation collapses to the original OV model. The same optimal velocity function for V_n^* is used as in the OV model. Compared with the original OV model, consideration of multi-vehicle interactions increases the extended model's stability.

Peng and Sun (2010) propose a similar extension for the FVD model. Neither Lenz et al. (1999) nor Peng and Sun (2010) consider driver reaction time. These two models were calibrated using numerical simulations; however, they have not yet been tested with real data.

2.5. Newell's simplified CF model and its extensions

Newell (2002) developed a parsimonious CF model following a very simple CF rule: the time-space trajectory of a vehicle in congested traffic on a homogenous highway is identical to the preceding vehicle's trajectory except for space and time shifts, as defined in Equation (21)

$$x_n(t+T) = \min\begin{cases} x_n(t) + uT & [\text{free} - \text{flow}] \\ x_{n-1}(t) - \delta & [\text{congestion}] \end{cases} \tag{21}$$

where $T = 1/(wk)$ is the wave trip time (or time shift) between two consecutive trajectories having w and k as the absolute values of wave speed and jam density respectively, $\delta = 1/k$ is jam spacing (or space shift), and $x_n(t+T)$ represents the longitudinal position of vehicle n at time $(t+T)$. Newell conjectures that the gap between two trajectories at time t depends on speed, and remains nearly constant if the highway is homogeneous. Newell further proposes that (T,δ) vary as if they were sampled independently from some joint probability distribution.

Besides its parsimoniousness (i.e., only two parameters T and δ are required), Newell's model has direct linkage to the macroscopic LWR theory (Lighthill and Whitham, 1955; Richards, 1956). Therefore, Newell's model is often adopted as the base theory in studying complex issues (Zheng et al., 2011a; Zheng et al., 2011b; Chen et al., 2012; Chen et al., 2012; Chen et al., 2014). For example, Zheng et al., (2013) use Newell's CF model to quantitatively measure the impact of lane-changing maneuvers on the immediately following vehicle.

Newell's CF model has also been extended to capture traffic oscillations. Oscillatory behaviors are generally caused by instabilities of the models. For example, in the stimulus-response- type models, instability arises when a following vehicle becomes highly sensitive to the preceding vehicle's stimulus (Herman et al., 1959). Newell's CF theory cannot be directly used for predicting characteristics of traffic oscillations because disturbances do not change in magnitude in this model due to the fact that a follower's trajectory is essentially replicated from the leader's by shifting in time and space. Thus, Laval and Leclercq (2010) relax the assumption of constant time shift (T) and make it time-dependent. By doing so, an oscillation can be interpreted as a deviation of T from the equilibrium T. They assume that, in congestion, deceleration waves can trigger some drivers (who are initially in equilibrium) to switch to "timid" or "aggressive" non-equilibrium modes. In their model, the trajectory of vehicle n is described as in Equation (22)

$$x_n(t) = \min\begin{cases} x_n(t-T) + \min\{uT, \tilde{x}_n(t)\} & [\text{free} - \text{flow}] \\ x_{n-1}(t - \eta_n(t)T) - \eta_n(t)\delta & [\text{congestion}] \end{cases} \tag{22}$$

where $\tilde{x}_n$ is the desired distance travelled by vehicle n during T, and $\eta_n(t)$ is a dimensionless variable introduced to capture deviations from Newell's model.

Chen et al. (2012) extended Laval and Leclercq's model, and developed a behavioral CF model based on empirical observations. They report that the model is capable of reproducing the spontaneous formation and ensuing propagation of stop-and-go waves in congested traffic.

2.6. Cellular Automata (CA) models

Cellular automata (CA) were historically proposed in the 1940s (Neumann, 1948) and popularized in the 1980s (Wolfram, 1983) to accurately reproduce macroscopic behavior of a complex system using minimal microscopic descriptions. A typical CA model constitutes four key components: the physical environment, the cells' states, the cells' neighborhoods, and local transition rules. The physical environment in which CA is applied for modeling traffic flow is obviously the road segment of interest, which consists of a one-dimensional lattice for a single-lane road. The lattice and the time are discretized into equal-length cells, typically equal to the vehicle length and the driver's average reaction time, respectively. The corresponding speed increment is computed as $\Delta x/\Delta t$. The state of each cell can be 0 (empty) or 1 (occupied), with two implicit assumptions: i) typically each cell is exactly occupied by one vehicle; and ii) drivers cannot react to any events between consecutive time steps (Zheng, 2014).

Nagel and Schreckenberg (1992) made the first notable contribution to the development of a CF model using cellular automata. They introduced a stochastic discrete CA model for freeway traffic. The road is discretized into cells of fixed width (7.5 meters in Nagel and Schreckenberg (1992)). At each time step, the model updates four consecutive steps, which are performed in parallel for all vehicles:

a. Acceleration: If the velocity V of a vehicle is lower than V_{max}, and if the distance to the next car is larger than V+1, the speed is increased by one [$V \rightarrow V+1$].
b. Deceleration: If a vehicle at cell i finds the next vehicle at cell $i+j$ (with $j \leq V$), it reduces its speed to j-1 [$V \rightarrow j-1$].
c. Randomization: With probability p, the non-zero velocity of each vehicle is decreased by one [$V \rightarrow V-1$].
d. Car motion: Each vehicle is advanced by V cells.

Although the discreteness of the model does not correspond directly to any property of real traffic, this simple model shows nontrivial and realistic behavior of traffic flow.

Krauss et al. (1996) argue that the discrete nature of the Nagel-Schreckenberg model hides many of its interesting features (for example, vehicle spacing cannot be less than the width of one cell, difficult to calibrate with real data etc.). Thus, they present a continuous version of the Nagel-Schreckenberg model, as shown in Equation (23).

$$\begin{aligned} &\tilde{V}_n(t+1) = \min\left[V_n(t) + a_{\max}, V_{\max}, S_{gap}(t)\right] \\ &V_n(t+1) = \max\left[0, \left(\tilde{V}_n(t+1) - b_{\max}, \eta_{ran,0,1}\right)\right] \\ &x_n(t+1) = x_n(t) + V_n(t+1) \end{aligned} \tag{23}$$

where $\tilde{V}_n$ is the desired speed, $a_{\max}$ is the maximum acceleration, $b_{\max}$ is the maximum deceleration, S_{gap} is the free space to the vehicle ahead, and $\eta_{ran,0,1}$ is a random number in the interval (0,1). Some randomness due to deceleration noise is considered when calculating the speed of the vehicle in each time step.

Krauss et al.'s (1996) continuous version of the Nagel-Schreckenberg model generates similar dynamics to those in the Nagel-Schreckenberg model except at high densities. Furthermore, unrealistic deceleration is observed because the safe velocity is calculated using the gap between two consecutive vehicles. To overcome this problem, Krauss and Wagner (1997) developed a model (known as S-K model), as shown in Equation (24)

$$\begin{aligned} &V_n^*(t+1) = \min\left[V_n(t) + a_{\max}, V_{\max}, V_{safe}\right] \\ &V_0(t+1) = V_n^*(t+1) - \epsilon\left(V_n^*(t+1) - \left(V_n(t) - b_{\max}\right)\right) \end{aligned} \tag{24}$$

$$V(t+1) = V_{ran,V_0,V_n^*}$$
$$x_n(t+1) = x_n(t) + V_n(t+1)$$

where V_n^* is the optimal velocity, V_{ran,V_0,V_n^*} is a random term between the optimal velocity and the deviation from the optimal velocity V_0, ε is the parameter determining the deviation from the optimal velocity, V_{safe} is a safe velocity below which no crashes are generated. The main difference between the Nagel-Schreckenberg model and the S-K model is that the S-K model calculates V_{safe} based on maximum allowable deceleration (as adopted from Gipps' model). It is reported that the S-K model outputs more realistic traffic characteristics at the macroscopic level. (For a detailed review of other CA-based CF models, see Maerivoet and De Moor, 2005.)

3. CAR-FOLLOWING MODELS: THE HUMAN PERSPECTIVE

The aforementioned Engineering CF models mostly focus on a driver's physical signals, rather than on their psychological reactions. Boer (1999) criticizes the inability of these models to explain human driving behaviors during CF. This is because they assume that: (i) drivers aim for optimal performance; (ii) driving is equivalent to the continuous application of a single control law; (iii) drivers use inputs that they may not be able to perceive, but are somehow able to compute; and that (iv) everything that cannot be explained by the model is noise, and can be attributed to perceptual and control limitations.

Most of the Engineering CF models provide no psychologically plausible characterization of how humans think about, and address, the driving problem. In normal and often complex driving situations, humans adopt strategies that are adequate rather than optimal because of their incomplete knowledge or insufficient time to evaluate all possible alternatives. If the current driving situation is acceptable, there is no reason to look for, and evaluate, alternatives; for example, if the speed is acceptable, there is no need to accelerate or waste resources to look for opportunities to overtake. This phenomenon contradicts traditional CF models where optimality requires that drivers expend all resources on trying to improve performance (Boer, 1999; Hancock, 1999). These criticisms of Engineering CF models are supported by the findings detailed below.

First, the surrounding environment plays an important role in close-following situations (such as urban areas and traffic congestion). In these situations, it is unlikely that drivers drive with the worst-case safety assumptions in mind. For example, despite the suggested minimum headway of 2 sec, 95.8% of drivers follow a headway less than 2 sec, and 47.9% have headways even less than 1 sec on the M27 motorway in UK (Brackstone et al., 2002). Similar situations have been observed on German freeways, where prevalent headways are 0.9 ~ 1 sec; in some instances, headways are found to be as low as 0.3 sec (Treiber et al., 2006). Research suggests that the surrounding environment (i.e. considering next-nearest neighboring vehicles, visual distractions, etc.) can have a significant influence on driver's confidence and driving behavior (Muhrer and Vollrath, 2011; Treiber et al., 2006). Therefore, the surrounding environment should be considered in CF models.

Second, each driver and driving style is different. Age and gender, for example, affect a driver in his/her perception of risky driving situations. A survey of drivers from Alabama, US, for example, shows that male teenagers engage more frequently in risky driving situations (e.g. close following, driving faster than the speed limit, etc.) than female adult drivers (Rhodes and Pivik, 2011). Ossen and Hoogendoorn (2011) found that considerable differences exist between the car-following behaviors of passenger car drivers. They observed clear differences in desired spacing and desired time headways among the drivers. Driver heterogeneity is also observed among car drivers and truck drivers where the latter group in general appears to drive with a more constant speed. Use of intelligent transportation systems and cooperative systems also influences driving styles (Farah et al., 2012).

Meanwhile, driving needs may also influence driving styles. Boer and Hoedemaeker (1998) categorize driving needs into 'motivational' and 'constraining' situations. Motivational driving involves situations such as the need to get somewhere fast or the enjoyment of high speed or pleasure (e.g., favoring certain routes, enjoying the surroundings), whereas constraining situations can be related to safety, workload, economic cost, social compliance and the need for comfort (in terms of acceleration and jerk).

Finally, a list of human factors based on the literature (e.g., Hamdar, 2012;Treiber and Kesting, 2013) is presented here:

a. Socio-economic characteristics (e.g., age, gender, income, education, family structure)
b. Reaction time
c. Estimation errors: Spacing and speeds can only be estimated with limited accuracy
d. Perception threshold: Human cannot perceive small changes in stimuli
e. Temporal anticipation: Drivers can predict traffic situation for the next few seconds
f. Spatial anticipation: Drivers consider the immediate preceding and further vehicles ahead
g. Context sensitivity: Traffic situation may affect driving style
h. Imperfect driving: For the same condition drivers may behave differently in different times
i. Aggressiveness or risk-taking propensity
j. Driving skills
k. Driving needs
l. Distraction
m. Desired speed
n. Desired spacing
o. Desired time headway

This section reviews the notable developments in attempts to incorporate these various human factors into the Engineering CF models.

3.1. Use of perceptual thresholds

Engineering CF models unrealistically assume that drivers can perceive and react even to small changes in the driving environment (for example, to slight change in speed difference or spacing). To overcome this problem, Wiedemann (1974) introduces the term 'perceptual threshold' to define the minimum value of the stimulus a driver can perceive and will react to. The models based on perceptual threshold are also known as 'psycho-physical' models. The threshold is expressed as a function of speed difference and spacing between the preceding and subject vehicles, and is different for acceleration and deceleration decisions. It increases driver alertness when spacing is small, and provides more freedom when it is large. An example of the distribution of the thresholds is shown in Figure 1. The thresholds are defined as:

AX The desired spacing between the front sides of two successive vehicles in a standing queue

BX The desired minimum following distance, which is a function of AX, the safety distance, and speed

SDV The action point where a driver consciously observes that he/she is approaching a slower leading vehicle; SDV increases with increasing speed difference

CLDV Closing delta velocity (CLDV) is an additional threshold that accounts for additional deceleration by the application of brakes

OPDV The action point where a driver notices that he/she is slower than the leading vehicle and starts to accelerate again

SDX A perception threshold to model the maximum following distance, which is approximately 1.5–2.5 times BX

Figure 1: Wiedemann's CF model (Source: Wiedemann, 1974)

The dark line in Figure 1 shows the decision path of an approaching vehicle. A vehicle travelling faster than the leader will get close to it until the deceleration perceptual threshold (SDV) is crossed (at Point A). The driver will then decelerate to match the leader's speed. However, as a human being, the driver is unable to accurately replicate the leader's speed, and spacing will increase until the acceleration perceptual threshold (OPDV) is reached (at Point B). The driver will again accelerate to match the leader's speed and the process continues, as shown in the unconscious reaction zone.

A modified version of the original Wiedemann model has been used in the commercial microsimulation software VISSIM (Fellendorf and Vortisch, 2010). Several calibration attempts for VISSIM model exist in the literature. For example, Park and Qi (2006) used Genetic Algorithm (Goldberg, 1989) to estimate model parameters; Gomes et al. (2004) manually calibrated four driver behavior parameters (among ten) while kept the others as default; Ištoka Otković et al. (2013) used neural network approach to calibrate the model parameters; and Lownes and Machemehl (2006) conducted sensitivity analysis of the simulation capacity output under various driver behavior parameters.

In a similar CF model by Fritzsche (1994), the CF plane is divided into five regions, as shown in Figure 2. For clarity, the figure is drawn for a CF case with two vehicles where the preceding vehicle is travelling at 20 m/s.

Figure 2: The CF phase diagram (source: Fritzsche, 1994)

PTN Perception Threshold Negative is the negative relative speed, i.e. $V_n > V_{n-1}$.

PTP Perception Threshold Positive is the positive relative speed, i.e. $V_n < V_{n-1}$.

AD Desired distance threshold represents a comfortable driving distance: $AD = A_0 + \tilde{T}.V_n$, where A_0 is the standstill distance from the leader and $\tilde{T}$ is the desired time headway.

AR Risky distance threshold is defined for conditions when spacing is too small for comfortable driving: $AR = A_0 + T_f.V_{n-1}$, where T_f is a fixed time headway with a magnitude of 0.5s.

AS Safety distance threshold represents situations when the follower realizes that he/she decelerates too much and reaches a safety distance with a positive speed difference. The follower then accelerates to match the leader's speed: $AS = A_0 + T_s.V_n$, where T_s

is the safe time headway, and is considered as 1s. The model requires that $\tilde{T} > T_s > T_f$.

AB Breaking distance threshold is an additional threshold applied to avoid collisions that might occur at high speeds.

These six thresholds divide the phase space into five regions: Danger, Closing in, Following I, Following II, and Free Driving. According to the model, a follower will decelerate only when he/she is in either 'Danger' or 'Closing in' regions.

Brockfield et al. (2004) presented a calibration attempt for Fritzche (1994) model with vehicle trajectory data using a gradient-free optimization method known as "downhill simplex" (Lagarias et al., 1998). However, the estimation results are not reported.

Fancher and Bareket (1998) propose an extension of the psycho-physical model (Wiedemann, 1974) by introducing a comfort zone which is used when a driver is within ±12% of the desired spacing. Being unable to perceive the speed difference relative to the leader, the driver will try to maintain the current speed in this zone. The free-flow zone (or no-reaction zone) is outside the comfort zone where the desired speed is maintained by the driver.

3.2. Driving by visual angle (DVA)

Michaels (1963) points out that visual extent or size of the preceding vehicle contributes to a driver's perception of the driving situation. Later, Gray and Regan (1998) show that human drivers are ill-suited to estimate longitudinal distances, absolute velocities, and accelerations of other objects in the scene. Rather, they are capable of accurately estimating time to collision (TTC) based on visual angles subtended by the preceding vehicle (that is, visual angle divided by rate of change of visual angle).

The basic assumption of the visual angle model is given by Michaels (1963) who states that when drivers are approaching a vehicle in front, they perceive the situation from the changes in the apparent size of the vehicle. More specifically, the relative speed is perceived through the changes in the visual angle subtended by the preceding vehicle. The visual angle (θ_n) can be calculated using Equation (25):

$$\theta_n(t) = 2\arctan\left(\frac{W}{2S_n(t)}\right) \approx \frac{W}{S_n(t)} \tag{25}$$

The angular velocity is found by differentiating this equation with respect to time *t*, as shown in Equation (26)

$$\frac{\mathrm{d}}{\mathrm{dt}}\theta_n(t) = -W\frac{\Delta V_n(t)}{(S_n(t))^2} \tag{26}$$

where W is the width of the preceding vehicle, S_n is the spacing between the preceding and the subject vehicles, measured from the front edge of the subject vehicle to the rear end of the preceding vehicle, and ΔV_n is the relative speed between the two vehicles.

Visual angle is used to replace relative spacing from the preceding vehicle, and angular velocity is used to replace relative velocity (or speed difference) in several Engineering CF models. As shown in Equation (27), Andersen and Sauer (2007) modified Helly's (1959) model by using visual angle as the stimuli. They call this model 'Driving by Visual Angle' (DVA)

$$a_n(t) = \alpha\left(\frac{1}{\theta_n(t)} - \frac{1}{\tilde{\theta}_n(t)}\right) + \lambda\frac{\mathrm{d}}{\mathrm{dt}}\theta_n(t) \tag{27}$$

where θ_n is the visual angle extent of the preceding vehicle; $\tilde{\theta}_n$ is the desired visual angle subtended by the preceding vehicle; $d\theta_n/dt$ is the rate of change in the visual angle; and α, λ are constants. The desired distance headway (or desired visual angle) should vary with speed, and is estimated by using the following formula

$$\tilde{\theta}_n(t) = 2\arctan\left(\frac{W}{\tilde{T}_n(t) \cdot V_n(t)}\right)$$

where $\tilde{T}_n$ is the desired time headway, and V_n is the speed of the subject vehicle. The simulation based on the DVA model produces similar speed and acceleration profiles, as observed from the actual driving situation. However, drivers' reaction time is ignored in the model, and a constant $\tilde{\theta}_n$ is used for simplicity in the simulation.

In a similar study, Jin et al. (2011) modified the full velocity difference (FVD) model (described in Section 2.4) using visual angle, as defined in Equation (28)

$$a_n(t) = \alpha[V_n^*(\theta_n(t)) - V_n(t)] - \lambda\frac{\mathrm{d}}{\mathrm{dt}}\theta_n(t) \tag{28}$$

where $d\theta_n/dt$ is the rate of change in the visual angle, α and λ are sensitivity coefficients. V_n^* is the optimal velocity a driver prefers based on the visual angle subtended by the preceding vehicle, and can be calculated as

$$V_n^*(\theta_n(t)) = V_1 + V_2\tanh(C_1 S_n(t) - C_2)$$

where S_n is the spacing between the two vehicles; and V_1, V_2, C_1, C_2 are parameters. Basically, this model is a conversion of the original FVD model, using visual angle. The authors have used the same parameter values to calculate V_n^* as were used in the FVD model.

Selecting an appropriate visual angle threshold, however, can be challenging. According to Michaels and Cozan (1963), the visual angle threshold ranges between 0.0003 to 0.001 rad/sec, with an average of 0.0006 rad/sec. If we consider a preceding vehicle's width of 1.8m and a speed difference of 10 km/hr, a threshold value of 0.0006 rad/sec indicates that a driver can detect a change in angular velocity subtended by the preceding vehicle when the relative spacing is less than 91 meters. Ferrari (1989) assumes a fixed angular velocity threshold (i.e., 0.0003 rad/sec), with the minimum time headway between two successive vehicles of 1sec, for his traffic simulation model. However, in a study of 60 drivers, Hoffmann and Mortimer (1996) found that subjects were not able to perceive the relative velocity or to make reasonable estimations of TTC if the angular velocity was less than 0.003 rad/sec.

3.3. Driver risk-taking, distraction, and error

Drivers' risk-taking behavior, distraction, and error in crash-prone and other extreme situations are probably the least explored topics in the CF modeling literature. In this section, notable efforts to consider these factors in CF modeling are reviewed.

3.3.1. Use of Prospect Theory to model risk-taking behavior

The cognitive process of driving in risk-taking situations involves perception, judgment and execution of a particular decision strategy (for example, braking or lane-changing). This process can be treated as a human decision-making problem where variables such as surrounding traffic, the environment, and the nature of the drivers themselves (of varying age, gender, driving experience, and risk attitude) are likely to affect driving choices.

The expected utility theory (Neumann and Morgenstern, 1949) for decisions under risk is the basis for modern decision-making theories. However, inconsistency between the actual decisions made and the decisions predicted by the utility theory led to the need to develop more realistic models to describe actual decision processes. In particular, prospect theory by Kahneman and Tversky (1979) is a well-accepted descriptive model that captures human decision making when there is the possibility of risky outcomes.

Hamdar et al. (2008) and Hamdar et al. (2014) develop a driver behavior model based on Kahneman and Tversky's (1979) prospect theory. Specifically, their model considers driving as a sequential risk-taking task. In their model, Kahneman and Tversky's prospect theory provides the theoretical and operational basis for weighing a driver's alternatives. The main variable of interest in the model is the subjective probability ($p_{n,i}$) of being involved in a rear-end collision with the preceding vehicle. This probability depends on acceleration, spacing, and speed difference, as shown in Equation (29)

$$p_{n,i} \approx p_n(t+\hat{\tau}_n) = \phi\left(\frac{\Delta V_n(t)\hat{\tau}_n + 0.5a_n(\hat{\tau}_n)^2 - S_n(t)}{\sigma(V_{n-1})\hat{\tau}_n}\right) \tag{29}$$

where $\hat{\tau}_n$ is the anticipation time span, S_n denotes spacing from the preceding vehicle, and $\phi(z)$ is a cumulative distribution function for the standardized Gaussian.

The gains (or losses) in this model are expressed in terms of increase (or decrease) in speed from the previous acceleration instance, and are constrained by the maximum desired speed of the driver and non-negativity of speed. The value function explaining the gain or loss using prospect theory is defined as in Equation (30)

$$U_{PT}(a_n) = x[w + 0.5(1-w)(\tanh(x)+1)](1+x^2)^{0.5(\gamma-1)} \tag{30}$$

where $x = a_n/a_0$; γ (non-negative) is the non-negative sensitivity parameter, a_0 is an acceleration normalizing factor (set to 1m/s^2), and w is the weight associated with negative acceleration. The driver sequentially evaluates candidate accelerations and eventually selects the one with the highest probability, using the following equation:

$$U(a_n) = \left(1-p_{n,i}\right)U_{PT}(a_n) - p_{n,i}w_c k(V_n, \Delta V_n) \tag{31}$$

If driver *n* decides to accelerate at instance *i*, he could increase speed (considered as gain) or be involved in a rear end collision (considered as loss) with a probability of $p_{n,i}$. The loss in a probable collision is assumed to be related to two terms: a seriousness term $k(V_n, \Delta V_n)$

representing the expected consequence if a collision had occurred, and a weighting factor w_c (a higher w_c corresponds with conservative drivers, and a lower w_c with aggressive drivers). Finally, to reflect the stochasticity in drivers' responses, the selected acceleration is retrieved from the following probability density function

$$f(a_n) = \begin{cases} \dfrac{\exp[\beta \times U(a_n)]}{\int_{a_{\min}}^{a_{\max}} \exp[\beta \times U(a')]da'} & a_{\min} \leq a_n \leq a_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

where parameter $\beta > 0$ reflects the sensitivity of choice to the utility $U(a_n)$. It can also account for the experience of the driver, i.e. a higher number for more experienced drivers reflect more stable driving style than the style of the least experienced driver.

The proposed model allows risk-taking maneuvers when drivers are uncertain of the leader's future behavior and, consequently, crashes are possible. Talebpour et al. (2011) later extended this model to consider surrounding traffic conditions (especially congested and uncongested situations). A driver can have different preferences, and hence different responses, to the same situation because of different surrounding traffic conditions. For example, in free-flow conditions, higher acceleration rates result in higher utilities; however, in congested traffic, the perceived pressure usually discourages drivers from accelerating. Therefore, two behavioral regimes are proposed, with two different utility functions, as indicated in Equation (33)

$$U_{PT}(a_n) = P(C) \cdot U_{PT}^{C}(a_n) + (1 - P(C)) \cdot U_{PT}^{UC}(a_n) \tag{33}$$

where *P(C)* denotes the probability of a driver being in a congested regime, and depends on several factors such as speed, average spacing and average speed difference between the subject vehicle and the preceding vehicles in all lanes, and the average spacing and average speed difference between the subject vehicle and the following vehicles in all lanes; U_{PT}^{C} and U_{PT}^{UC} are utility functions for congested and uncongested traffic conditions respectively. The model was calibrated using Next-Generation Simulation (NGSIM) data (Alexiadis et al., 2004). The calibrated model shows consistency with observed phenomena in real traffic – phenomena such as: the probability of high acceleration rates decreases with an increase in density; higher spacing leads to higher acceleration rates; the higher the speed, the more a driver desires to reduce speed; and, in a congested situation, drivers maintain a speed closer to the average speed of the surrounding vehicles to avoid a crash.

3.3.2. CF models which consider driver error and distraction

Human drivers are prone to making driving errors, which are responsible for crash in most cases. 'Human error' is a broad term that has been used rather loosely to encompass almost all the unsafe acts that lead to crashes. Reason (1990) classifies unsafe acts into two distinct classes of behavior: errors and violations. An 'error' can be defined as the failure of planned actions to achieve the desired outcome, whereas a 'violation' is the deliberate infringement of some regulated or socially accepted code of behavior (Parker et al., 1995). Violation can be committed for a variety of reasons and can be distinguished through the issue of intentionality. Parker et al. (1995) found that the tendency to commit driving violations is a positive predictor of crash involvement, whereas no link between error-proneness and crash involvement was found. Stanton and Salmon (2009) further categorize driver errors into five groups: action errors, cognitive and decision-making errors, observation errors, information retrieval errors, and violations. CF can be affected by any of these errors; however, how and to what extent it is affected remains elusive and requires future research. This review focuses on driver errors – especially those caused by distractions.

'Driver distraction' can be defined as a diversion of attention away from activities critical for safe driving to a competing activity (Lee et al., 2008). 'Distraction' is also described as multi-task driving which reduces attention to driving itself. Studies have shown that multitasking while driving deteriorates driving performance, increases reaction time, and impacts lateral lane position and vision. This, in turn, poses serious safety hazards on the roads where 10% to 80% of reported crashes are related to distracted driving (McEvoy and Stevenson, 2007; Przybyla et al., 2012; Stutts, 2003). In a recent review of driver distraction, Young and Salmon (2012) explain how distraction could be responsible, at least to some extent, for most driver-related errors.

A major limitation of Engineering CF models is that they are designed to produce crash-free environments for the convenience of microscopic traffic simulations. However, crash-free environments are not always desirable, for example, for the study of extreme situations in safety analysis, and for the measurement of the effectiveness of in-vehicle active safety technology. Hamdar and Mahmassani (2008) explored six well-known Engineering CF models to observe their behaviors in crash-prone situations by relaxing their safety constraints. They simulated 3600 vehicles on a 10 km highway in a 2-hour period, and their findings are summarized in Table 1 (below).

Table 1: Summary of findings of six Engineering CF models after relaxing safety constraints (Source: Hamdar and Mahmassani, 2008)

With these modifications, the Wiedemann, Gipps and CA models showed more stable behavior compared to the GHR, S-K and IDM/IDMM models, although the number of crashes is unrealistically high. These findings call for a richer representation of the cognitive process in the Engineering CF models, in order to produce realistic crash-causing behavior.

To more effectively incorporate human behavioral considerations into Engineering CF models, Van Winsum (1999) extended Helly's (1959) desired spacing model. The proposed model captures human behavior through the desired time headway, assuming that there could be substantial differences in the desired time headway between drivers that reflect variables such as driving conditions and mental effort. For example, less skilled drivers generally choose to drive with larger time headways to avoid collisions. Heino (1996) found that a driver's mental effort increases (as indicated by a reduction in heart rate variability) when the time headway is smaller than the preferred one. Van Winsum (1999) modified the desired spacing in Helly's model as

$$\widetilde{\Delta X}_n = \tilde{T}_n \cdot V_n$$

where $\tilde{T}_n$ denotes the desired time headway, which can be influenced by visual conditions (such as fog, rain and night driving), driver state (such as fatigue and inebriation), and the mental effort deployed in following the preceding vehicle. When the distance to the preceding vehicle is smaller than desired, the driver is assumed to decelerate until $\widetilde{D}_n$ is reached. Van Winsum (1999) also shows that, in response to the preceding vehicle's deceleration, the subject vehicle decelerates with a rate as shown in Equation (34)

$$b_n = \alpha.e.\left[\Delta X_n / \sqrt{2b_{n-1}\left(\widetilde{\Delta X}_n - \Delta X_n\right)}\right]^f + \beta + \varepsilon \tag{34}$$

where b_{n-1} is the deceleration of the preceding vehicle; ϵ is a random error term, and α, β, e, f are parameters. The use of the preceding vehicle's deceleration can be problematic and is rare in the CF literature because it is very difficult for the driver to measure it. Rather, Gipps (1981) uses the driver's estimated deceleration of the preceding vehicle. The model only covers the negative acceleration of the driver. An acceleration algorithm for the model is proposed by Wang et al. (2011), and is shown in Equation (35)

$$a_n = \alpha\left(\Delta X_n / \widetilde{\Delta X}_n\right) + \beta(\Delta V_n) + \lambda + \varepsilon \tag{35}$$

where α, β are constants; λ represents the influence of driving purpose and driving habit; other variables are the same as those for Equation (34). However, acceleration's direct dependency on distance can lead to unrealistic acceleration rates. The model has not been tested using real data.

Treiber et al. (2006) point out that the majority of Engineering CF models (such as OVM, FVD and IDM) produce unrealistic dynamics and crashes during simulation. Therefore, they compensate for the destabilizing effects of reaction times and estimation errors (in ΔV, TTC) by considering the spatial and temporal anticipations of the driver. More specifically, Treiber et al. (2006) propose four extensions to IDM: finite reaction times, estimation errors, spatial anticipation, and temporal anticipation. They call their model the 'Human Driver (meta-) Model' (HDM). In this model, the driver is aware of the surrounding traffic environment and can modify their driving behavior accordingly.

Przybyla et al. (2012) extend Newell's (2002) simplified CF model to accommodate the impact of distractions on driving. They assume that the distracted driver continues to drive at the constant speed (attained in the previous time step) throughout the distracted event. Their model divides the driver's trajectory into two types: the trajectory followed by a perfect driver (in other words, a perfect follower who can be described by Newell's model), and the trajectory followed by a distracted driver. However, they further assume that the driver is either distracted or not distracted for the entire trajectory. This could be problematic in representing actual behavior.

Bevrani and Chung (2012) improve Gipps' (1981) model by considering human imperfection in processing information and executing actions. More specifically, they include human perception limitations in detecting speed differences, extra delay in driving phase changes (assuming that reaction time increases after being in a fixed situation; that is, either in a constant speed or in an acceleration phase), and driver imperfection in adjusting speeds. However, human errors, such as distraction and risk taking, are omitted in their model.

An error-able CF model is proposed by Yang and Peng (2010). For the evaluation of active safety technologies (AST), they propose a stochastic CF model with an error mechanism derived from the Road-Departure Crash-Warning System Field Operational Test (RDCW), a large-scale naturalistic driving database. The model calculates the desired acceleration of the driver as a function of following distance, speed difference, and/or time headway. It also considers uncertainties in calculating the final acceleration, assuming that when the following distance is large, the driver cannot perceive accurately and has more room to deviate. The Yang and Peng (2010) model is represented by Equation (36)

$$\begin{aligned} \tilde{a}_n(t) &= f_{\tilde{a}_n}\left(\Delta X_n(t), \Delta V_n(t), T_n\right) \\ a_n(t) &= f\left(\tilde{a}_n(t), \sigma\right) \end{aligned} \tag{36}$$

where $\tilde{a}_n$ is the desired acceleration, and σ captures the deviation. The model's parameters are calculated from the RDCW database. Three major types of driving errors are introduced: perceptual limitation, time delay, and distraction. The human perception limitation is implemented based on the same method as the one described in Section 3.1: the introduction of the minimum threshold of speed difference that a driver can detect and will respond to. Time delay is estimated through a recursive least square identification process, and distraction is identified based on the statistical analysis of the RDCW data. The frequency and duration of distraction are also estimated. During distraction, the model continues to use the information from the previous time step without updating it.

4. CONCLUSIONS AND DISCUSSIONS

This paper presents a review of the state-of-the-art of CF modeling from two different perspectives: the engineering perspective and the human factor perspective. Representative models of each perspective have been reviewed. The main features of these models (including their strength and weakness) are also summarized in Table 2 and Table 3, respectively. Compared with previous reviews of CF models, the paper is unique in that it provides a comprehensive review of notable attempts to incorporate human-factors in CF models through various approaches, such as visual angle-based models, and models that consider driver risk taking, distraction, and driver errors.

Table 2 Representative CF models: the engineering perspective

Table 3 Representative CF models: the human factor perspective

This review is an important step in advancing CF modeling, as the disregard of human factors (such as perceptional limitation, risk-taking behavior, error, and distraction) in the current CF models means that they are unrealistically over-simplified. Overall, the main limitation of the Engineering CF models is that they do not reflect the psychologically plausible characterization of how humans think about, and accomplish, driving tasks. For example, they do not capture the interdependencies among the decisions made by the same driver over time, or the effect of the surrounding environment (such as visibility and surrounding vehicle dynamics). The models represent instantaneous decision-making, which underestimates a driver's planning and anticipation capabilities, while overestimating their ability to evaluate all possible alternatives and to achieve an optimal level of driving performance. This, in turn, means that they are unsuited to the investigation of important issues which demand fine representations of driver behaviors. These issues include the analysis of crash-prone traffic conditions; the understanding of widely-reported puzzling phenomena such as capacity drop, stop-and-go oscillations, and traffic hysteresis; the microscopic analysis of traffic dynamics; and the development and evaluation of advanced vehicle control and safety systems.

Note that there are many (commercial or free) microscopic simulation software packages available based on various CF theories. For a detail review on popular microscopic simulation packages, see Barceló (2010). Although some of these simulation packages attempted to account for human behavior features (e.g., a reaction time distribution and perceptual thresholds are used in VISSIM and PARAMICS), many human factors which are crucial for describing human car-following (CF) behavior are, by and large, ignored (e.g., driving error, distraction, and risk-taking behavior).

To conclude this paper, common issues and research needs (in the authors' opinion) in data collection, model development, model calibration and validation in modeling CF are summarized below.

***Data collection**:* Fully incorporating human factors into the Engineering CF models pose challenges in data collection. The primary data source used for developing CF models is loop detector data or trajectories at best, which can only provide basic vehicular information. Driver characteristics, which are critical for deciphering drivers' thinking processes during the CF procedure, cannot be extracted from this type of data. This serious data limitation often leads to the fact that human factors are usually over-simplified in the few CF models that indeed considered human factors. These models relied on only one or two parameters to indirectly capture the total impact of drivers' individual characteristics and cognitive features. Examples of these parameters are: perceptual thresholds, reaction time, visual angle, maximum desired speed, desired time headway, and etc. The model parameters related to human factors in most cases are unobservable in nature and, hence, are difficult to calibrate and validate using mainstream traffic data, which often leads to a further simplification of assuming these parameters to be constant across individuals ignoring driver heterogeneity. In our view, to obtain these model parameters, innovative data collection methods aiming to capture drivers' psychological disposition, perceptional performance, and cognitive function during CF are needed. For example, reaction time in different car-following circumstances can be observed from experiments using advanced driving simulator (see Haque and Washington (2014) as an example). Other human factors may also be obtained (completely or partially) by using driving simulator and/or from real driving experiments with instrumented vehicle. Of course, drivers in traffic flow may behave differently from what is observed from these experiments. Undesirable impact of such discrepancy can be minimized by employing advanced data analysis techniques. Unfortunately, in our extensive literature review we observed very few experiments designed for obtaining human factors critical for car-following modelling. More work in this regard is clearly needed. To get around the issue of a lack of human data, two common practices are: a) vehicle trajectory data are used to estimate some of these human factors (e.g., Brockfield et al., 2004; Park and Qi, 2006) with optimization technique; or even worse, b) values from the human factor literature or simply based on common sense are applied.

Model development: Overall, human-factor-oriented CF models are comparatively few in the literature, while Engineering CF models are predominant. Some recent advances in CF modeling attempt to enhance the Engineering CF models by incorporating a few human psychological characteristics. However, future research on this front is in great need in this regard because many important psychological factors are still missing from these models (for example, error-able CF, distractions, driving needs, and interaction with other vehicles). To develop humanlike CF models, it is necessary to obtain a common understanding of the problem by seamlessly integrating the latest advances from both the Engineering and the human-factor-oriented CF models, bridging their gaps, and reconciling their inconsistencies. While a number of different psychological parameters are suggested by various researchers, no studies have ranked their importance in describing driver behavior in the CF situation, or attempted to accurately quantify their values. Consequently, many of the reported models simply take psychological parameters from the human factor literature without validating them within the context of CF. Meanwhile, although the need for incorporating human factors into CF models is great, adding these factors can dramatically increase the model's complexity, which underscores the importance of maintaining the balance between maximizing the model's predictive and explanatory power and minimizing the model's complexity. As recommended in Zheng (2014), factors considered in the model need to be behaviorally, empirically, and statistically justified for the target driver population. Another important and often-ignored issue in developing CF models is that the CF model should be able to be easily integrated into mainstream lane change modeling frameworks to provide a complete description of vehicular movements on road.

Calibration and validation: CF models often contain a wide range of variables, posing a significant challenge for model calibration and validation. Discussions on calibrating CF models are scattered in the literature (e.g., Brockfeld et al., 2004; Kesting and Treiber, 2008; Ossen and Hoogendoorn, 2008; Hoogendoorn and Hoogendoorn, 2010), however, guidance on the systematic and rigorous calibration and validation of traffic flow models is still lacking. The majority of the models were tested either numerically or by matching certain macroscopic traffic flow features (which, strictly speaking, can only invalidate microscopic CF models). This free-style approach causes substantial confusions, even cherry picking. In our view, a bi-level evaluation strategy should be generally preferred in developing a new CF model: at the macroscopic level, the model should be capable of explaining widely-observed traffic flow characteristics; at the microscopic level, vehicular movements should be close to actual observations (e.g., trajectories, speed profile, and acceleration profile). Furthermore, similar to lane changing models (Zheng 2014), vehicular data used for calibrating and validating CF models were mostly collected in developed countries where drivers are generally less aggressive than their counterparts in developing countries. To capture the full spectrum of CF, it is desirable to use data containing more diverse driving behaviors, particularly more aggressive driving behavior. Finally, calibrating and validating CF models containing human factors are even more challenging because of the difficulty in measuring these human factors, which often forces researchers to (over-)simplify the representations of the human factors in calibrating CF models, as discussed previously.

In summary, an improved and more comprehensive representation of human factors in CF models can lead to the next breakthrough in modeling vehicular movement on roadways. This comprehensive literature review of the state-of-the-art in the research field of human factor CF modeling is highly significant in providing a comprehensive knowledge base for this future work.

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APPENDIX: NOTATIONS

a_n	Acceleration applied by driver *n* (positive or negative)
$\tilde{a}_n$	Desired acceleration of driver *n*
a_{comf}	The comfortable acceleration/deceleration
$a_{\max}$	Maximum acceleration/deceleration
b_n	Deceleration of driver *n*
b_{n-1}	Deceleration of driver *n-1*
$\tilde{b}_n$	Desired deceleration of driver *n*
$\hat{b}$	An estimate of the deceleration applied by the preceding vehicle
$b_{\max}$	Maximum acceleration
V_n	Speed of subject vehicle
V_n^*	Optimal velocity
$\tilde{V}_n$	Desired speed
ΔV_n	Speed difference between the subject vehicle the preceding vehicle $(V_{n-1} - V_n)$
$V_{\max}$	Maximum velocity
V_{safe}	Safe velocity for a vehicle
V_0, V_1, V_2, C_1, C_2	Constants
x_n, x_{n-1}	Position of vehicle *n* and *n-1* respectively
ΔX_n	Spacing from preceding vehicle: $\Delta X_n = x_{n-1} - x_n$
$\widetilde{\Delta X}_n$	Desired following distance
L_{n-1}	Length of the preceding vehicle
s_{n-1}	The effective length of vehicle *n*-1 (L_{n-1} + safety gap)
S_{gap}	Safety gap between two vehicle
S_n	Spacing between two vehicles measured from the front edge of the subject vehicle to The rear end of the preceding vehicle: $S_n = \Delta X_n - L_{n-1}$
$\tilde{S}_n$	Desired S_n
W	Width of the preceding vehicle
S_{jam}, S_1	Vehicle spacing at standstill situation
d	Constant which represents minimum spacing
t	Time
τ_n	Reaction time
$\hat{\tau}_n$	Time span for a decision
T_n, T	Time headway, Time shift
$\tilde{T}_n$	Desired time headway
$p_{n,i}$	Probability of being involved in a rear-end collision with the preceding vehicle
$\phi(z)$	A tabulated cumulative distribution function for the standardized Gaussian.
$M(.)$	Represents a memory function
$H(.)$	Heaviside function with a value of either 0 or 1
h_n	Headway
h_n^*	Critical headway
$\varepsilon_n^{cf}, \varepsilon_n^{ff}$	Normally distributed error terms for car-following and free-following
$\eta_{ran,0,1}$	Random normal distribution with mean 0 and standard deviation 1
$\alpha, \beta, \gamma, \lambda, w$	Parameters
δ	Jam spacing
k_n	Density of traffic ahead
φ	Gradient difference in a sag
θ_n	Visual angle subtended by the preceding vehicle
$\tilde{\theta}_n$	Desired visual angle subtended by the preceding vehicle
$d\theta_n/dt$	The rate of change in the visual angle

Figure

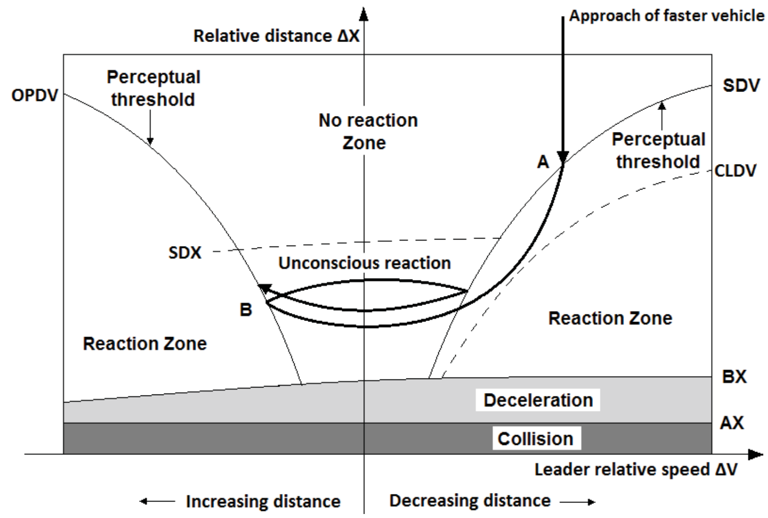

Figure 1: Wiedemann's CF model (Source: Wiedemann, 1974)

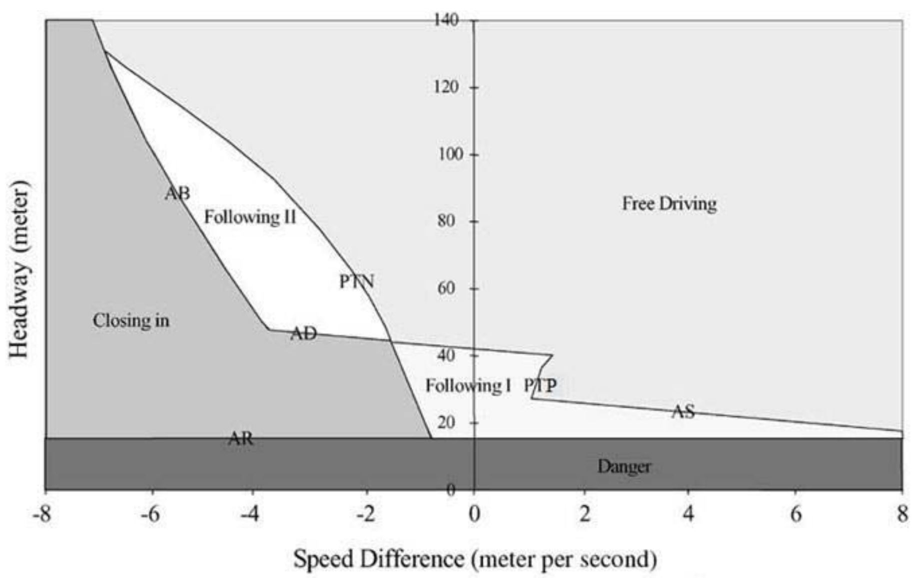

Figure 2: The CF phase diagram (source: Fritzsche, 1994)

Table

Table 1 Summary of findings of six Engineering CF models after relaxing safety constraints (Source: Hamdar and Mahmassani, 2008)

Model	Modification of the safety constraint	Result
GHR model	The sensitivity term λ is treated as a random variable with a normal distribution ($\lambda_{mean} = C/\Delta X_n$; λ_{std} =0.1, where C is a constant). However, this modification alone did not cause any crashes. Crashes were created when ΔV_n was treated as a normally distributed random variable with mean as ΔV_n, and standard deviation of 0.5.	A complete flow break-down with the occurrence of 561 crashes
Gipps' model	Gipps' model has a safety constraint $x_{n-1} - s_{n-1} > x_n$, where s_{n-1} is the safety distance. A normally distributed random risk term D_n is subtracted from s_{n-1} so that the safety distance can be negative to allow crashes to occur.	The normally distributed random risk term $D_n$with mean 0.1 and std 0.1 created 42 crashes.
Continuous version of CA model (Krauss et al., 1996)	The safety constraint is relaxed by allowing $V_{max} = s_{gap}$, and by allowing speed to be equal to $s_{gap} + 0.1$meter.	29 crashes were produced. Unrealistically high deceleration rates were observed.
S-K model	V_{safe} in the S-K model is increased by 0.27 m/s; however, no crashes were generated until V_{safe} was increased to 0.45m/s.	A total of 2013 chain type crashes occurred, and occupied most of the 10 km highway.
IDM and IDMM	In the IDM model, the last term in the desired spacing $\frac{V_n(t)\Delta V_n(t)}{2\sqrt{a_{\max} a_{\mathrm{comf}}}}$ creates the safety buffer. The safety buffer was removed to create crashes.	A complete traffic breakdown with 1211 crashes for IDM and 674 crashes for IDMM were observed.
Wiedmann model	The emergency braking mode is used to prevent crashes. This mode was replaced by a normal mode of deceleration, and the safety constraint was removed from the desired spacing threshold (BX) to generate crashes.	17 chain-type crashes were observed.

Table 2 Representative CF models: the engineering perspective

Model category	Model name (developers)	Model Equation	Strengths	Weakness and comments	Human factors[1] included
GHR model and their extensions	Linear CF model (Chandler et al., 1958)	$a_n(t) = \lambda.\Delta V_n(t-\tau_n)$	• Simplest model • The stability of the model is proved • Several functional form of λ is found, however the authors used $\lambda = a$ as a constant.	• The model is too simple to describe actual traffic phenomena as the later models do.	• Driver reaction time
	Non-linear GHR model (Gazis et al. (1961)	$a_n(t) = \alpha V_n(t)^\beta \frac{\Delta V_n(t-\tau_n)}{\Delta X_n(t-\tau_n)^\gamma}$	• Simple and well-established model • Most studied model • Driver reaction time is considered • Model parameters can be easily estimated from either vehicle trajectory data or macroscopic data (using speed-density relationship) • Many estimations of the parameters are available	• Use of identical reaction time for all drivers does not capture inter-driver heterogeneity. • Human ability to perceive small changes in driving conditions are overestimated. • Model parameters do not consider behavioral differences between acceleration and deceleration. • The model is highly sensitive to velocity difference. When velocity difference is zero, any value of spacing is acceptable.	• Driver reaction time
	Lee (1966)	$a_n(t) = \int_0^t M(t-s)\Delta V_n(s)ds$	• Introduces memory function in the linear CF model. • The memory function makes the driver decisions consistent with the past driving profile, rather than creating instantaneous accelerations. • Removes unrealistic peaks in acceleration profile of the driver.	• The implementation of the model in traffic simulation is considerably more complex due to the need of maintaining an array of past conditions for each vehicle.	• Driver reaction time • Memory function to consider past driving experience

[1] Literature shows that human drivers are ill-suited to estimate longitudinal distance to preceding vehicle, absolute velocities, and accelerations of other objects in the scene (Gray and Regan, 1998). Therefore, these terms are omitted from human factors along with the speed and acceleration of the subject vehicle (these are elements of laws of motion). Similarly, maximum acceleration and deceleration are omitted as they are related to the subject vehicle's capability. All the other parameters that are to some extent related to human driving behavior are reported in this 'human factors' column.

Model category	Model name (developers)	Model Equation	Strengths	Weakness and comments	Human factors[1] included
	Ahmed (1999)	See Equation (4)	• The model considers acceleration/deceleration asymmetry. • Two separate models are proposed for free flow and CF, separated by a headway threshold. • Traffic density is considered within 100m in front of the driver. • Driver heterogeneity is considered by distributing the reaction time over the driver population.	• Transition from free flow to CF and vice-versa are not smooth. • Human ability to perceive small changes in driving conditions are overestimated.	• Distribution of Driver reaction time to consider driver heterogeneity • Acceleration/Deceleration asymmetry • Driving condition in terms of traffic density
	Herman and Rothery (1965)	$a_n(t) = \sum_{i=1}^{m} \alpha_i \Delta V_{n,n-1}(t - \tau_n)$	• The model considers multi-vehicle interactions where driver follows more than one preceding vehicle.	• Linear CF model is used as a base model which has already been criticized for its simplicity	• Driver reaction time • Multi-vehicle interaction
Desired measures models	Helly's model (Helly, 1959)	$a_n(t) = \alpha_1 \Delta V_n(t - \tau_n) + \alpha_2 [\Delta X_n(t - \tau_n) - \widetilde{\Delta X}_n(t)]$	• The desired space headway creates a safety buffer which prevents collision. • The desired space headway is dependent on speed and acceleration of the subject vehicle.	• Direct dependency on following distance might create unrealistically high acceleration/deceleration. • Model parameters for desired space headway are difficult to estimate as it is unobservable in usual traffic data. • Desired space headway is not driver dependent.	• Driver reaction time • Desired space headway
	Koshi et al. (1992) and Xing (1995)	$a_n(t) = \alpha_1 \frac{\Delta V_n(t-\tau_1)}{\Delta X_n(t-\tau_1)^l} + \alpha_2 \frac{[\Delta X_n(t-\tau_2) - \widetilde{\Delta X}_n(t)]}{\Delta X_n(t-\tau_2)^m} - \gamma \sin\varphi + \lambda[\tilde{V}_n - V_n(t - \tau_3)]$	• Direct dependency on following distance is solved. • The model has four stages: standard driving, acceleration from standing queue, effect of gradient and free flow acceleration.	• While the physical condition of the road in terms of gradient is considered in this model, horizontal curvature effect is neglected. • No estimation effort is found for this model.	• Driver reaction time • Desired space headway • Desired speed

Model category	Model name (developers)	Model Equation	Strengths	Weakness and comments	Human factors[1] included
	IDM (Treiber et al., 2000) IDMM (Treiber and Helbing, 2003) HDM (Treiber et al., 2006)	$a_n(t) = a_{\max}^{(n)}\left[1-\left(\frac{V_n(t)}{\tilde{V}_n(t)}\right)^{\beta} - \left(\frac{\tilde{S}_n(t)}{S_n(t)}\right)^2\right]$	• This model considers both the desired speed and the desired space headway. • The desired space headway depends on speed, speed difference, minimum spacing, maximum acceleration, comfortable deceleration and desired time headway. • The model considers vehicle capacity. • Several attempts are found for calibration of the model. • The model is a combination of free-flow and CF model. • The transition between free-flow and CF model is smooth.	• Reaction time is ignored in IDM. • Two extensions of IDM: IDMM and HDM are available. • IDMM considers driver's adaptation capability with surrounding environment to improve desired time headway calculation. • In HDM four extensions to IDM is proposed: finite reaction times, estimation errors, spatial anticipation, and temporal anticipation.	IDM and IDMM • Desired space headway • Desired speed • Comfortable deceleration • Desired time headway HDM • Driver reaction time • TTC estimation error • Multi-vehicle interaction • Relative distance error
Safety distance models	Kometani and Sasaki (1959)	$\Delta X_n(t-\tau_n) = \alpha V_{n-1}^2(t-\tau_n) + \beta V_n^2(t) + \gamma V_n(t) + d$	• The model seeks to specify a safe following distance. • The model parameters are estimated. • The parameter d prevents the model from collision.	• Does not describe stimulus-response type function as most of the other models.	• Driver reaction time
	Newell (1961)	$V_n(t) = V_{\max}\left[1-\exp\left(\frac{-\lambda(\Delta X_n(t-\tau_n)+d)}{V_{\max}}\right)\right]$	• It is a non-linear model and follows stimulus-response type function.	• Direct dependency on density might result in unrealistic accelerations or decelerations.	• Driver reaction time
	Gipps (1981)	Equation (13)	• The model use a safety distance which prevents collision • Has separate model for free-flow and CF • The transition between free-flow and CF is smooth unless the preceding vehicle brakes harder than anticipated, preceding vehicle changes the lane or a new vehicle enters in front of the subject vehicle from adjacent lane. • The model offers distribution of behavioral parameters to offer driver heterogeneity.	• Although many behavioral parameters used, the model does not consider driver errors. • How the parameter values are estimated/selected is not explained	• Driver reaction time • Desired acceleration • Desired deceleration • Estimation of preceding vehicle's deceleration • Desired speed

Model category	Model name (developers)	Model Equation	Strengths	Weakness and comments	Human factors[1] included
			• (Vehicle size + safety distance) follows a normal distribution.		
Optimal velocity model	OV model (Bando et al., 1995) (Bando et al. (1998)	$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right]$ $a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t - \tau_n)\right) - V_n(t - \tau_n)\right]$	• The optimal velocity (OV) depends on the distance from the preceding vehicle. • The model was estimated by Helbing and Tilch (1998).	• Driver reaction time is ignored in this version but later introduced in Bando et al. (1998). • The model produces unrealistic acceleration and decelerations. • The model is unrealistically sensitive to reaction time.	• Driver reaction time
	GF model (Helbing and Tilch (1998)	$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right] + \lambda\left(\Delta V_n(t)\right) \cdot H\left(-\Delta V_n(t)\right)$	• Heaviside function (*H*) works at negative velocity difference and solves the problem of unrealistic deceleration that occurs in OV model. • Model parameters are estimated from real data.	• The model still produces unrealistic accelerations. • Driver reaction time is ignored.	NA
	FVD model (Jiang et al., 2001)	$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right] + \lambda\left(\Delta V_n(t)\right)$	• Velocity difference is included explicitly to overcome unrealistic accelerations/decelerations	• Having a single parameter for both acceleration and deceleration might lead to an unrealistic situation where the subject vehicle brakes insufficiently, even if the distance to the preceding vehicle is extremely short. • Driver reaction time is ignored • No estimation of the model parameters are found, neither the model is applied on real data.	NA
	AFVD model (Gong et al., 2008)	$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t)\right) - V_n(t)\right] + \lambda_1\left(\Delta V_n(t)\right) \cdot H\left(-\Delta V_n(t)\right) + \lambda_2\left(\Delta V_n(t)\right) \cdot H\left(\Delta V_n(t)\right)$	• The model uses different responses in acceleration and deceleration as an improvement over FVD model.	• AFVD model takes longer time then FVD model to become stable. • Numerical simulation is done but the model is not yet applied on real data and the parameters are not estimated.	NA

Model category	Model name (developers)	Model Equation	Strengths	Weakness and comments	Human factors[1] included
	Lenz et al. (1999)	$a_n(t) = \sum_{i=1}^{m} \alpha_i \left[V_n^*\left(\frac{\Delta X_{n,n-i}(t)}{i}\right) - V_n(t)\right]$	• The model considers multi-vehicle interactions which increases the model's stability.	• Driver reaction time is ignored • Numerical stability analysis is performed, but the model parameters are not estimated from real data.	NA
	Davis (2003)	$a_n(t) = \alpha\left[V_n^*\left(\Delta X_n(t-\tau_n) + \tau_n \Delta V_n(t-\tau_n)\right) - V_n(t)\right]$	• The OV function is extended to consider the change in velocity difference as well as headway. • Unrealistic sensitivity to reaction time is solved.	• Although the OV function is measured at time $(t-\tau_n)$, the velocity is measured at time t, which needs a behavioral justification.	• Driver reaction time
Newell's simplified CF model	Newell (2002)	$x_n(t+T) = \min\begin{cases} x_n(t) + Tu, \\ x_{n-1}(t) - \delta \end{cases}$	• It is a parsimonious model (only two parameters are required). • The model can express macroscopic flow theory very well. • The model has been extended to capture traffic oscillation.	• The model is purely based on traffic flow theory. • No driver behavior parameters are available.	NA
Cellular Automata (CA) models	Nagel and Schreckenberg (1992) Krauss et al. (1996)	Equation (23)	• Randomness in speed is implemented to accommodate deceleration noise • The discrete version by Nagel and Schreckenberg (1992) was difficult to calibrate with real data, hence the continuous version is proposed by Krauss et al. (1996) as shown in the equation presented in this table.	• Unrealistic deceleration is observed at high densities. • Driver reaction time is ignored. • Although the discreteness of the model does not correspond directly to any property of real traffic, the model shows nontrivial and realistic behavior of traffic flow at the macroscopic level.	NA
	S-K model (Krauss and Wagner, 1997)	Equation (24)	• Unrealistic deceleration problem of Krauss et al.'s (1996) model is solved by replacing S_{gap} with V_{safe}. • V_{safe} is calculated based on maximum allowable deceleration. • S-K model outputs more realistic traffic characteristics at the macroscopic level than Krauss et al.'s (1996).	• Driver reaction time is ignored	NA

Table 3 Representative CF models: the human factor perspective

Model category	Model name (developers)	Model Equation	Strengths	Weakness and comments	Human factors included
Use of perceptual thresholds	Wiedemann (1974) Fritzsche (1994)	NA	• Perceptual thresholds selects minimum value of the stimulus a driver can perceive and will react to. • The thresholds are expressed as a function of speed difference and relative spacing. • They are different for acceleration and deceleration decisions. • The thresholds divides the driving plane to several decision zones, such as 'no reaction zone' (free-flow), 'closing in', 'danger' (must decelerate), and 'car-following'.	• The thresholds are simply obtained from the human factors literature. • The equations for different thresholds are undisclosed.	• Perceptual thresholds
Driving by visual angle	DVA model (Andersen and Sauer, 2007)	$a_n(t) = \alpha\left(\frac{1}{\theta_n(t)} - \frac{1}{\tilde{\theta}_n(t)}\right) + \lambda\frac{\mathrm{d}}{\mathrm{dt}}\theta_n(t)$	• Human driver are capable of accurately estimating time to collision (TTC) based on visual angles subtended by the preceding vehicle. • Visual angle is used to replace relative spacing from the preceding vehicle, and angular velocity is used to replace relative velocity. • DVA model produces similar speed and acceleration profiles, as observed from the actual driving situation	• Driver reaction time is ignored. • A constant value for desired velocity is used for simplicity; thus, this model ignores driver heterogeneity.	• Visual angle • Angular velocity
	Jin et al. (2011)	$a_n(t) = \alpha[V_n^*(\theta_n(t)) - V_n(t)] - \lambda\frac{\mathrm{d}}{\mathrm{dt}}\theta_n(t)$	• It is the FVD model using visual angle.	• No estimation is done for this model; the authors used same parameter values as used for FVD model. • Driver reaction time is ignored	• Visual angle • Angular velocity

Model category	Model name (developers)	Model Equation	Strengths	Weakness and comments	Human factors included
Use of Prospect Theory to model risk-taking behavior	Hamdar et al. (2008)	Equation (29), (30), (31) and (32)	• The subjective probability of being involved in a rear-end collision depends on acceleration, spacing and speed difference. • The gains (or losses) in this model are expressed in terms of increase (or decrease) in speed from the previous acceleration instance. • Final acceleration is retrieved from a probability density function to reflect stochasticity in driver's response. • The model allows risk-taking maneuvers when drivers are uncertain of the leader's future behavior and, consequently, crashes are possible.	• The probabilistic nature may create more acceleration noise than other models such as GHR, Gipps and IDM.	• Risk-taking behavior • Maximum desired speed • Anticipation time • Uncertainties of the preceding vehicle's speed • Uncertainties of the spacing • Random components of the subjective utility function • Reaction time (not explicitly included)
CF models which consider driver error and distraction	Van Winsum (1999)	$b_n =$ $\alpha . e . \left[\Delta X_n / \sqrt{2b_{n-1}(\widetilde{\Delta X}_n - \Delta X_n)}\right]^f +$ $\beta + \varepsilon$ $\widetilde{\Delta X}_n = \tilde{T}_n \cdot V_n$	• The model assumes that the driving conditions and mental effort can make substantial difference in desired time headway. • The desired time headway can be influenced by visual conditions (such as fog, rain and night driving), driver state (such as fatigue and inebriation), and the mental effort deployed in following the preceding vehicle.	• The model use preceding vehicle's deceleration as a parameter which is really difficult to measure accurately for a human driver. Rather, Gipps (1998) uses an estimate to preceding vehicle's deceleration. • The model parameters are not estimated • The model is not yet tested with real data	• Desired time headway • Driver reaction time • Driving condition
	Yang and Peng (2010)	$\tilde{a}_n(t) = f_{\tilde{a}_n}\left(\Delta X_n(t), \Delta V_n(t), T_n\right),$ $a_n(t) = f\left(\tilde{a}_n(t), \sigma\right).$	• It is a stochastic CF model. • Driver error mechanism is developed from a large scale naturalistic driving database • Three major types of driver errors are introduced: perceptual limitation, time delay, and distraction.	• The effect of distraction is hypothesized as no change in driving condition during distracted period. No real experiment is done to prove the assumption. • Homogenous driving population used	• Driver reaction time • Driver distraction • Perceptual thresholds • Stochastic error behavior