Microscopic Modelling of Area-Based Heterogeneous Traffic Flow: Area Selection and Vehicle Movement

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Abstract

The area-based (i.e., non-lane based) heterogeneous traffic (as in developing countries) has significant differences from the lane-based homogeneous traffic (as in developed countries). In area-based traffic, drivers generally ignore the lane markings and perceive the entire road space while progressing longitudinally. The traditional car-following and lane changing models are not directly applicable to model such driving behaviour.

This research aims to microscopically model the dynamics of the subject vehicle in area-based traffic flow. The modelling is conducted in two steps: area selection, and vehicle movement. In step-1, a multinomial logit (MNL) model is considered for selecting the area-based movement direction of the subject vehicle. The choice space of the subject vehicle is divided into numbers of realistic radial cones considering the possible moving directions of the subject vehicle in the next simulation time step. These radial cones form the alternatives for the decision of the subject vehicle. The attributes of the alternatives are defined in terms of angular deviation from the direction of the flow, spacing and relative speed. This model is calibrated and validated for both cars and motorcycles using real trajectory data, and the results demonstrate the good performance of the model. In step-2, a modified intelligent driving model (MIDM) is proposed to simulate the next position of the subject vehicle (along the selected direction as the outcome of the model in step-1). The parameters of the MIDM model are calibrated using real data for cars and motorcycles.

The deterministic parameters for MNL model and the empirical distribution of the MIDM model parameters are utilized to stochastically simulate vehicle trajectories with the initial and boundary conditions determined by the real dataset. The comparison of the macroscopic properties between the simulated and real datasets provides promising results for the simulation applicability of the proposed modelling framework.

Keywords: Area-based traffic, non-lane based traffic, Angle-based discretization, Confusion matrix, MNL, Microscopic modelling, MIDM, Visual perception area, ROC space

1 Introduction

A high percentage of passenger cars on urban roads can be considered as homogeneous with respect to the vehicle type (herein referred to as homogeneous, Figure 1(a)). The heterogeneous traffic with respect to vehicle type (herein referred to as heterogeneous) is composed of multiple vehicle types with different behavioural and mechanical characteristics. This includes passenger cars, motorcycles, auto-rickshaws, heavy vehicles such as buses, trucks and light commercial vehicles. In developing countries (such as India) traffic is heterogeneous and drivers generally ignore the lane marking. Here, drivers perceive the entire road space for their movement and such traffic movement is termed as area-based traffic (Figure 1(b)). The resulting traffic flow is significantly different from lane-based traffic (Figure 1(a)) where drivers maintain the lane marking rules while driving.
Figure 1 A schematic diagram for traffic flow differentiating (a) lane-based, from (b) area-based (a rectangular block represents a vehicle and arrow indicates the direction of traffic flow).

For microscopic modelling a large number of car-following (CF) and lane changing models (LC) are proposed in literature. Interested reader should refer to (Saifuzzaman and Zheng (2014) and Brackstone and McDonald (1999) for a review on CF models, and Zheng (2014) for a review on LC models. These models are the building blocks of microscopic traffic simulation packages such as Aimsun (Barceló (2010)) and Vissim (PTV (2011)) and are limited to lane based traffic flow conditions only. Addressing the need for microscopic modelling for traffic not respecting the lane marking, recently several attempts have been undertaken to develop area-based heterogeneous traffic flow models.

The driving behaviour of area-based flow involves frequent lateral movements while progressing longitudinally. The lateral movement of individual vehicles basically generate the angular deviation from the direction of the flow. This is analogous to the pedestrian walking behaviour, though the manoeuvre is limited by the mechanical characteristics of the vehicle. In literature, the discrete choice framework is proposed by Antonini, Bierlaire, and Weber (2006) to model the pedestrian dynamics. Motivated by which, this research aims to develop a parsimonious modelling framework to capture the driving behaviour of area-based traffic flow. We propose a two-step hierarchical modelling framework where a discrete choice framework in step-1 is developed to identify the direction in which the vehicle can move. The second step of vehicle movement specify how far the vehicle will move along the direction selected in step-1. The combined effect of these two steps defines the trajectory of the vehicle.

Discrete choice modelling theory has long been utilised in number of engineering and behavioural applications. The uniqueness of its applications in literature is the way the alternatives and their attributes are defined. The novelty of our research compared to the discrete choice framework proposed by Antonini et al. (2006) includes:

a) The logical procedure to define the number of alternatives and the attributes of the alternatives for direction selection in area-based traffic (in step -1); and

b) Modelling the vehicular movement in step-2.

The rest of the paper is organized as follows: the next section reviews the literature for modelling driving behaviour for area-based traffic. Section 3 presents the proposed modelling framework; Section 4 describes the dataset; Section 5 focuses on the area selection model calibration and validation while Section 6 on the vehicle following model calibration. Section 7 discusses on entire model validation from simulation of the model. Finally, the paper is concluded in section 8 with the future research direction.

2 Literature review on area-based modelling

A common approach in the literature for modelling area-based heterogeneous traffic is the application of a cellular automata (CA) based model. Different researchers, Lan and Chang
(2005), Mathew, Gundaliya, and Dhandra (2006), and Mallikarjuna and Rao (2009) have proposed modifications to traditional CA model, ‘NaSch model’, from Nagel and Schreckenberg (1992) for its application in area-based traffic.

Lan and Chang (2005) have proposed to consider small cell size and allow vehicles to occupy multiple cells based on its physical dimension. This avoids unrealistic speed jumps and provides flexibility to consider vehicles with different dimensions. Mathew et al. (2006) have proposed to determine cell size based on the passenger car equivalent (PCE) value of vehicles. Mallikarjuna and Rao (2009) have proposed to consider cell size consistent with the length of the smallest vehicle. Nevertheless, the modelling requires appropriate CF and LC logics to change the position of vehicles over the spatial temporal cell grids. These logics need further development and validation to capture complex behaviour of different vehicle types in area-based traffic flow.

Hossain and McDonald (1998) proposed a simulation framework where a) the vehicle position is defined in a coordinate system that eases the identification of the nearby vehicles; and b) the CARSIM model (see Benekohal and Treiterer (1988) for details) is used for simulating the longitudinal stimulus-response CF model. Here, the stimulus is defined by the front vehicle in the neighbourhood. The lateral movement is modelled considering the nearest side vehicle but the details of the logical rules for such movement are not provided in the paper.

Arasan and Koshy (2005) presented a general simulation framework, for a mid-block section, which includes vehicle generation, placement and movement. Here, the vehicles are considered as rectangular blocks. The applicability of the simulation model is cross-checked by headway distribution and speeds of the different vehicle classes. An analogous micro-simulation framework of Arasan and Koshy (2005) has been developed by Krishnamurthy and Thamizh Arasan (2014) that incorporates road width and traffic volume. The simulation was used to quantify PCE values. Their study indicates that PCE value increases with an increase in the width of road space and varies significantly accordingly to change in the ratio of volume and capacity.

Lee, Polak, and Bell (2009) proposed mathematical models for movements of motorcycle in mixed traffic flow. The one-on-one vehicle interaction was modelled by two mathematical models such as longitudinal headway model and oblique and lateral headway model. The multivehicle interaction was modelled by a path choice model. A car-following model was developed to calculate the longitudinal headway. The oblique or lateral headway model describes the motorcycle safety distance when the motorcyclist follows a vehicle obliquely or laterally. A regression model was used to calculate the oblique or lateral headway. Finally, a path choice behaviour for the motorcycle was described using a multinomial logit model. All of these models were calibrated on vehicle trajectory data.

The strip-based space discretization framework is proposed by Mathew, Munigety, and Bajpai (2013) for modelling the driving behaviours of non-lane based mixed traffic. Here, the lane is divided into a fixed number of strips. Each vehicle can occupy more than one strip based on its physical dimension. The longitudinal movement of vehicles is considered along with the strips. The subject vehicle is influenced by multiple leaders with different vehicle types. The lateral movement is modelled by defining very small strip widths which allows tactical overtaking manoeuvres over multiple strip changes. The lane changing model (Ehmanns (2001)) was modified to consider multiple strips and vehicles move laterally one strip at a time. This model is analogous to traditional lane-based traffic flow when the strip and the lane are of equal width. The methodology was implemented in the open source traffic simulator SUMO (Behrisch, Bieker, Erdmann, and Krajzewicz (2011)) and the developed tool is termed as SiMTraM (Simulation of Mixed Traffic Mobility).
Choudhury and Islam (2016) proposed an acceleration model for weak lane discipline. The model was developed based on latent leader and acceleration components. The acceleration of a subject vehicle was modelled using a two-level structure. In first level, a random utility based discrete choice model was used to calculate the probability of any front vehicle being the governing leader. In second level, the acceleration component was modelled using a stimulus-response based nonlinear general motor (GM) model.

Nair, Mahmassani, and Miller-Hooks (2011) have proposed a porous flow approach where the area-based system is considered as a disordered system. The classical equilibrium speed-density relation (i.e., linear) is modified by considering a pore space distribution for such systems. The traffic stream at the microscopic level is disordered and defines a pore space which is modelled by different probability distribution functions. The system is described by the set of state variables, where a) the traffic density is measured as area density that considers vehicular concentration per unit area for each vehicle class; b) the traffic flow is measured in vehicles per second for a unit cross-section of the roadway; and c) the speed is assumed as constant for each vehicle class. The continuity equation is defined for each vehicle class and finite difference scheme is adapted to show the evolution of the traffic stream in two different vehicle classes. Modelling pore spaces are generally complicated due to the size variations and different manoeuvrability of vehicles.

Comprehensive review of the microscopic modelling is outside the scope of this paper. Interested readers are referred to Mumigety and Mathew (2016) for a detailed review of microscopic modelling in area-based traffic.

To summarize, researchers have developed different approaches such as CA, strip-based, porous flow etc. for area-based traffic flow modelling. The CA model results are dependent on the rules that should incorporate the behaviour. The strip-based approach is a simplification of lane-based simulation with virtual lanes termed as strips. The porous flow approach is analogous to granular flow through a porous medium which relies on different pore sizes and pore space distributions is complicated for area-based traffic. It can be concluded that the research on area-based modelling is still in early stage and there are significant avenues for model development and more realistic representation of the driving behaviour.

3 Proposed modeling framework

In area-based traffic the individual driver constantly looks for a suitable gap to move forward at the desired speed. This behaviour results in frequent changes in the vehicles’ lateral position while progressing longitudinally. The movement of the vehicle can be classified into unconstrained and constrained driving pattern.

The unconstrained pattern is the free flow condition, where the presence of other vehicles on road does not influence the driving behaviour of the subject vehicles. Modelling such behaviour is straightforward and simple kinematic vehicular equations can be applied for vehicle movement. Such pattern is not the focus area of this research.

Under constrained pattern, the driving behaviour of the subject vehicle is influenced by the presence of other vehicles. Such behaviour is the focus of this research.

The proposed modelling framework predicts the movement of a vehicle (subject vehicle) at time $t + \Delta t$ given the dynamics of all the vehicles until time $t$. Here, $\Delta t$, is the discrete time step for the modelling. The dynamics of the vehicles within the visual perception area of the subject vehicle define the variability of the state of traffic (speed and density) being experienced by the subject vehicle. Such variability influences the lateral and longitudinal
movements of the subject vehicles i.e., follow the current direction of motion or move laterally. This research proposes to capture such behaviour through following hierarchical steps:

a) **Step-1: Area selection:** Here, the subject vehicle evaluates the traffic within its perception area and adjusts the need and direction for the area-based movement; and

b) **Step-2: Vehicle movement:** Here, the movement of the vehicle along the direction selected in the first step is modelled.

The sequential application of the two steps defines the trajectory of the vehicle. The details of each steps are provided in the following sub section.

### 3.1 Step-1: Area selection

Here, a discrete choice modelling framework with the following tasks is proposed:

a) Defining the alternatives (choice set). Details in section 3.1.1;

b) Defining the attributes of the alternatives. Details in section 3.1.2; and

c) Modelling the selection of an alternative. Details in section 3.1.3

#### 3.1.1 Defining the alternatives (choice set)

The choice space for the movement of the subject vehicle (S) is defined by its driver’s visual perception area within the ‘macular’ peripheral vision (Strasburger, Rentschler, and Jüttner (2011)). This space is divided into ‘N’ number of realistic radial cones (see Figure 2). The direction of the traffic flow is considered as the reference direction. The angular bound of the \(n^{th}\) alternative is expressed as \( (\theta_{n-1}, \theta_n) \), where the variable \( \theta \) is the angle measured from the reference direction as illustrated in Figure 2. In general, the direction of the \(n^{th}\) alternative is \( \left[ \theta_n - \frac{(\theta_n - \theta_{n-1})}{2} \right] \), except for the extreme alternatives (1\(^{st}\) and \(N^{th}\) alternative) where the direction can be modified considering the data availability.

Effectively, the vehicle should be traveling in the direction of the traffic flow (longitudinal direction), though lateral movement is required to gain speed or for turning at the next road section. A driver can scan both the left and the right side to seek lateral and longitudinal movement opportunities. Thus, the alternatives are considered symmetrical along the direction of the traffic flow, which implies that the total number of alternatives, \(N\), is always an odd number. Here, the direction of \(\left(\frac{N+1}{2}\right)^{th}\) alternative is along the direction of the traffic flow.

In practice, there can be multiple options for the driver and in our proposed modelling framework, there is no limit on the value of \(N\). Higher the value of \(N\), more refined is the model in the selection of the movement direction. However, increasing \(N\) increases the size of model (section 3.1.3), which further requires huge amount of data (with sufficient spread) to reasonably calibrate a large model. As a trade-off between accuracy and complexity with large size model, we propose to set the \(N\) for different vehicle classes considering the vehicles’ manoeuvre flexibility and data availability. Higher manoeuvres flexibility vehicles (such as motorcycle) can have larger value of \(N\) compared to that of lower manoeuvre flexibility vehicles (such as car).

The angle for the central (along direction of traffic flow) alternative is considered close to the human central vision (when eye focuses straight ahead). To be conservative it can be modelled as \(2^\circ\). The other alternatives are symmetrical along the centre direction and are to be defined considering the data availability. For the current research real vehicle trajectory data (details in section 4) is used to specify the alternatives, for cars and motorcycles, the details for which are presented below:
Figure 2 An illustration for defining the finite number (N) of alternatives in the choice space of subject vehicle (S).

For cars, the alternatives are restricted within the muscular peripheral vision of -9° to 9°. Three number of alternatives (N = 3: n = 1 for right, n = 2 for centre, and n = 3 for left) are specified. The centre direction of 2° [-1° to 1°] is considered. The right and left direction is defined symmetrical around the centre direction with the bounds of [-1° to -9°] and [1° to 9°], respectively. Figure 3 illustrates these alternatives where, S represents the subject vehicle and L₁ and L₂ are two potential lead vehicles in the centre and left alternative, respectively.

Figure 3 The angular bounds for three alternatives based on ‘macular’ peripheral vision from −9° to +9°.

For motorcycles, the observed movement direction in our data set is restricted between -11° to +15°. However, the sample size for the movement between 3° to 15° and -3° to -11° is low (less than 6% of the data). The region of [-11° to +15°] is divided into five choice sets (N = 5). The centre direction of 2° [-1° to 1°] is considered. The direction for other four choice sets is defined considering cumulative deviations of 2° from the centre direction. Figure 4 illustrates the five alternatives where, S represents the subject vehicle. Note, here the direction of alternative 1 and alternative 5 is not the centre of the respective cone. The direction of - 4° for alternative 1 and 4° for alternative 5 is considered because majority of movement along 1st and 5th alternative was observed along that direction.
The choice set of a subject vehicle as a motorcycle and defined angular bounds for five alternatives based on angle deviation observed from aforementioned data.

### 3.1.2 Defining the attributes of alternatives

The attributes of the alternatives should influence the behaviour of the subject vehicle in selecting the alternative. The dataset available for the current research is from a basic uninterrupted road segment (details in section 4), which is outside the influence of the merging and diverging movements from on and off ramps. Therefore, for the current research, the attributes for the area selection are only defined by the parameters that influence driver’s behaviour under basic uninterrupted road segment. This include *Angular deviation*, *Spacing* and *Relative speed*, the details for which are outlined below. Note: the modelling framework can be easily extended to consider additional attributed such as driver’s necessity towards a particular lateral position (for example, to exit from the road or take a turn at the next intersection etc.).

**Angular deviation (AngDev):** It is the absolute deviation of the direction of the alternative from the direction of the traffic flow. As mentioned earlier, the reference direction is the direction of the traffic flow. Therefore, the angular deviation of an alternative is the absolute magnitude of the direction of the alternative. The behaviour of the subject vehicle in selecting an alternative should be influenced by its respective angular deviation. The higher the deviation, the less attractive the alternative should be. For example, in Figure 5a, the three possible directions for subject vehicle as a car along the right ($\theta_R$), centre($\theta_C$), and left ($\theta_L$) for the next time step are defined with the current direction ($\theta_S$). The reference direction is the direction of the traffic stream, which is also the direction of the centre alternative ($\theta_C = 0$). The angle deviation for the subject vehicle along the centre, left, and right alternatives is defined as $\Delta \theta_C = |\theta_S - \theta_C|$; $\Delta \theta_L = |\theta_L - \theta_S|$; $\Delta \theta_R = |\theta_R - \theta_S|$ in degrees, respectively.

**Spacing:** This is the spatial gap between the subject vehicle and a potential lead vehicle (the one nearest to the subject vehicle) present in the alternative. It is expected that lower spacing is perceived as constrained driving by the subject vehicle, and an alternative with lower spacing can therefore be less attractive than one with higher spacing. An alternative is either occupied by potential leaders or empty.

- If an alternative is empty and the lateral movement of the subject vehicle is unconstrained by the physical boundary of the road, then the spacing for the alternative
should be modelled as default perceived spacing \((g_o)\) for the subject vehicle. Here, \(g_o\) is a calibration parameter.

- The spacing for an occupied alternative is measured considering the vector projection of the leader position along the direction of an alternative. In Figure 5b, the left alternative is occupied by a potential leader (L) and vector \(r\) denotes the vector position of the leader with respect to subject vehicle (S). The spacing for S along this alternative is calculated from the vector projection \(r \cos \Delta \theta\) where \(\Delta \theta\) is the angle between position vector \(r\) and the direction of alternative \(\theta_L\); that is, \(\Delta \theta = \delta - \theta_L\), where angle \(\delta = \tan^{-1}\left(\frac{\Delta Y}{\Delta X}\right)\). \(\Delta X\) and \(\Delta Y\) are the longitudinal, and the lateral distances between the L and S, respectively.

Therefore, spacing for a subject vehicle is defined by Equation (1), as follows:

\[
Spacing, s = \begin{cases} 
  r \cos \Delta \theta & \text{if alternative is occupied} \\
  g_o & \text{if alternative is empty}
\end{cases}
\]

\(1\)

**Relative speed \((Rspeed)\):** This is the speed difference between the perceived speed along the alternative direction and the current speed for a subject vehicle projected along the alternative direction.

- The perceived speed along the alternative direction for an occupied alternative is the vector projection of the speed of the lead vehicle (from the alternative) along the direction of the alternative. For example, in Figure 5c the speed of the L in an alternative with direction \(\theta_L\) is presented as vector \(v_L\). The L has a longitudinal and lateral speed of \(v_{L,X}\) and \(v_{L,Y}\), respectively. The direction of vector \(v_L\) with respect to the reference direction (direction of flow) is expressed as \(\psi_L\) and is calculated as \(\tan^{-1}\left(\frac{v_{L,Y}}{v_{L,X}}\right)\). The vector projection of \(v_L\) along the direction of the alternative is expressed as \(v_L \cos \Delta \psi_L\), where \(\Delta \psi_L = \psi_L - \theta_L\). Similarly, the vector projection of the subject vehicle along the direction of the alternative is defined by \(v_S \cos \Delta \psi_s\), where the angle \(\psi_s\) is \(\tan^{-1}\left(\frac{v_{S,Y}}{v_{S,X}}\right)\) and \(v_{S,X}\) and \(v_{S,Y}\) are the longitudinal and lateral speeds for the subject vehicle, respectively.

- For an empty alternative, the perceived speed along the alternative direction is a calibration parameter and is expressed as the desired speed \((v_d)\) for the subject vehicle.

The relative speed of a subject vehicle is defined by Equation (2), as follows:

\[
Rspeed, \Delta v = \begin{cases} 
  v_L \cos \Delta \psi_L - v_S \cos \Delta \psi_s & \text{if alternative is occupied} \\
  v_d - v_S \cos \Delta \psi_s & \text{if alternative is empty}
\end{cases}
\]

\(2\)

**What about the alternatives which are constrained by the physical road boundary?**

An alternative can have restricted movement due to physical boundary of the road. For instance, when the lateral distance of the subject vehicle from the left (right) edge of the road is less than 1 meter, we define the subject vehicle is close to the left (right) physical boundary of the road and its lateral movement along the left (right) direction is restricted by that road physical boundary. To capture this affect, the default spacing for the left (right) alternative is considered as 1 meter and the default desired speed \((v_d)\) is considered as zero. This significantly reduces the attractiveness of the alternative.
3.1.3 Modelling the selection of an alternative

The utility of each alternative is expressed as a linear combination of its attributes, as defined in Equation (3).

\[ U(n) = \lambda_n + \sum_{k=1}^{N} \beta_{n,k} X_{n,k} \quad (3) \]

where \( k = 1,2, \ldots, N \) and \( N \) represents the finite numbers of alternatives for \( S \). For the current research, \( N = 3 \) for cars and \( N = 5 \) for motorcycles.

\( X_{n,k} \) is a vector of \( k \) attributes describing alternative \( n \). For the current research, attributes include Spacing, Rspeed and AngDev.

Figure 5(a) Angle deviations for a subject vehicle, (b) spacing for a subject vehicle, (c) relative speed for a subject vehicle.
\( \beta_{n,k} \) is a vector of parameters for \( k \) attributes describing alternative \( n \); and 
\( \lambda_n \) is a scalar parameter. The parameter \( \lambda_n \) links to the deterministic term with unobservable term for alternative \( n \) and is known as an alternative specific constant (ASC).

An alternative prediction can be modelled by using either a multinomial logit (MNL) model (under multivariate extreme value distribution of unobserved effects in utility) or a multinomial probit (MNP) model (under multivariate normal distribution of unobserved effects in utility). Interested readers should refer to Luce (2005) for a detailed description of a logit model and Hensher, Rose, and Greene (2005) for details of a probit model.

In the MNL, the probability of selecting a particular alternative is calculated using Equation (4).

\[
P(n|C) = \frac{e^{\lambda_n + \sum_{k=1}^{N} \beta_{n,k} x_{n,k}}}{\sum_{j} e^{\lambda_j + \sum_{k=1}^{N} \beta_{j,k} x_{j,k}}}
\]  

(4)

where \( P(n|C) \) represents the probability of selecting an alternative \( n \) in a given choice set \( C \).

In the MNP, the probability of selecting a particular alternative is calculated using Equation (5).

\[
P(n|C) = \int_{V_i-V_j}^{\infty} \int_{V_i-V_j}^{\infty} \ldots \int_{V_i-V_j}^{\infty} \varphi(w_1, w_2, \ldots, w_n/\Sigma_j) \, dw_1 \, dw_2 \ldots \, dw_n
\]  

(5)

where \( P(n|C) \) represents the probability of selecting an alternative \( n \) in a given choice set \( C \), \( \varphi(\cdot) \) denotes the multivariate normal density with a zero-mean vector, a covariance matrix \( \Sigma_j \) and \( w_n = \varepsilon_i - \varepsilon_j \) is the difference for unobserved effects in utility. If there is a correlation between the alternatives, the MNP model should then be considered instead of the MNL model.

Maximum likelihood estimation is adopted to estimate the optimal values of the model parameters. For the MNL model, the log-likelihood function with the choice probability defined in Equation (4) is globally concave in parameters \( \beta_{n,k} \), which helps in the numerical maximisation procedures. For the MNP model, the multivariate normal integral (Equation (5)) needs to be computed. This practical complication is addressed using the GHK simulator, where the integrals are approximated using the Monte Carlo simulation. This calculation is extremely time consuming. In this research, NLOGIT (2012) is used to estimate the parameters for these models.

In this research, MNL model is considered because: (a) the estimates from the MNP model showed weak correlation between alternatives (see Appendix A for details of the estimated results from the MNP model); and (b) MNL model is simple to implement and has been widely and successfully used in the literature to model discrete choices.

3.2 Step-2: Vehicle movement

The direction of the selected alternative at time \( t \) defines the direction of the movement of the subject vehicle at time \( t + \Delta t \). The spacing and relative speed parameters of the selected alternative are used as an input to simulate vehicle movement. For this a modified version of the Intelligent Driver Model (termed here as MIDM) is proposed.
Traditional, Intelligent Driver Model (IDM) (see Treiber, Hennecke, and Helbing (2000) for details) is widely used in literature. Equation (6), presents the IDM’s CF acceleration ($a_s$) of the subject vehicle.

$$a_s = a_{max} \left[ 1 - \left( \frac{v}{v_d} \right)^\delta - \left( \frac{s^*(v, \Delta v)}{s} \right)^2 \right]$$  \hspace{1cm} (6)

where $a_{max}$ is the maximum acceleration of vehicle (S), $v_d$ is its desired speed, $s$ is the spacing between subject and lead vehicles measured from the front bumper of a subject vehicle to the rear end of a leading vehicle, $s^*$ is the desired minimum gap, and $\delta$ is an acceleration exponent (model parameter). The desired space headway expressed in Equation (7) is dependent on the speed of the subject vehicle, relative speed, the minimum spacing ($s_0$) at the jam situation (standstill), the maximum acceleration of the subject vehicle, a comfortable deceleration ($b$), and the desired time headway ($T$).

$$s^*(v, \Delta v) = s_0 + s_1 \frac{v}{v_d} + \frac{v\Delta v}{2\sqrt{a_{max}b}}$$  \hspace{1cm} (7)

The minimum spacing $s_0$ in congested traffic is significant for low-speed vehicles only; the term $s_1$ represents nonlinear jam distance, $T$ for safety time headway; $b$ for desired deceleration.

For MIDM, the spacing ($s$) and relative speed ($\Delta v$) parameters of the original IDM are modified considering the vector projection of the relative position (refer to Equation (1)) and the relative vehicle speed (refer to Equation (2)) along the direction of the selected alternative (in step-1). The modified IDM (MIDM) equation is expressed as:

$$a_s = a_{max} \left[ 1 - \left( \frac{v_S \cos \Delta \psi_S}{v_d} \right)^\delta - \left( \frac{s^*(v_S \cos \Delta \psi_S, \Delta v)}{s} \right)^2 \right]$$  \hspace{1cm} (8)

where,

$$s^* = s_0 + s_1 \frac{v_S \cos \Delta \psi_S}{v_d} + v_S \cos \Delta \psi_S T + \frac{v_S \cos \Delta \psi_S \Delta v}{2\sqrt{a_{max}b}}$$  \hspace{1cm} (9)

Here, $v_d, a_{max}, b, T, s_0, s_1, \text{ and } \delta$ are MIDM parameters need to calibrate on vehicle trajectory data.

### 4 Real vehicle trajectory dataset

Vehicles trajectories from an urban midblock road section (Maraimalai Adigalar Bridge on Chennai-Nagapattinam Highway in Saidapet) in Chennai, India collected by Kanagaraj, Asaithambi, Toledo, and Lee (2015) is used in this research. The data is freely available from the link presented in their paper. Interested readers are referred to the paper for details. Here, the main information is presented below:

The study section is on a bridge with six-lane separated uninterrupted road (3 lanes in each direction). Here, major merging or diverging is more than a km upstream or downstream of the study section. The data are from one direction (northbound approach) only. The dimensions of
the study section are illustrated in Figure 6. The data were collected using video cameras that were placed on the roof of a building neighbouring to the section. The 30 minutes vehicle trajectory data were extracted from 2:45 pm to 3:15 pm that exhibits area-based vehicle dynamics. The trajectories were extracted from the video sequence at a time resolution of 0.5 seconds.

![Figure 6 Schematic diagram of data collection site in Chennai, India (adapted from Kanagaraj et al. (2015)).](image)

The collected datasets included 3,005 vehicle trajectories and classified into passenger cars, motorcycles, auto-rickshaws, and heavy vehicles (buses, trucks, and light commercial vehicles (LCV)). Approximately 56.4% were motorcycles, 26.6% were passenger cars, 12.2% were auto-rickshaws, and the remaining 4.8% were heavy vehicles.

Here, 70% of the raw data (termed as training or calibration data) are used for area selection model calibration and the remaining 30% (termed as testing or validation data) are used to measure the performance of the area-selection model. The vehicle movement model is calibrated on 130 cars and 40 motorcycles randomly selected from the data.

The training data are used to develop a database for individual car and motorcycle with the respective alternative and attributes to calibrate the area selection model. Each time step information is presented by one block having three or five rows for individual subject vehicle as car or motorcycle, respectively. Table 1 shows the sample of choice data format in the econometric software NLOGIT for the individual subject vehicle. The variable name “Alt” is an accounting index that informed the NLOGIT which alternative is assigned to a line of data. The variable “Cset”, which is repeated in each row of data in the choice set, provided the number of choices in each choice set. The “choice” of “1” means that an alternative is selected, and ‘0’ means otherwise. The last three variables: Spacing (m), Rspeed (m/s), and AngDev (degree) represent the spacing, relative speed, and angle deviation, respectively. For occupied alternatives, the spacing is calculated based on the proposed methodology.

For the empty alternative, the default value of the spacing ($g_0$) is calibrated on observed spacing (in training dataset) within defined visual perception area. The centre alternative (i.e., alternative 2 for car and alternative 3 for motorcycle) is calibrated considering the observed 50th percentiles of the observed spacing within the 30m front bounded visual perception area. The other alternatives are calibrated considering the observed 70th percentiles of the observed spacing within the 30m front bounded visual perception area. The calibrated default spacing is 20.53m for Centre and 25.64m for other alternatives.
In addition, for the empty alternative, the perceived speed (here, known as desired speed ($v_d$) for the subject vehicle) along the alternative direction is calibrated considering the average traffic speed within 100m longitudinal bounds and its calibrated default desired speed is 6.47(m/s).

Table 1 A sample of choice data format in NLOGIT for individual subject vehicle

<table>
<thead>
<tr>
<th>Vehicle Class</th>
<th>Alternative Name</th>
<th>Alt</th>
<th>Cset</th>
<th>Choice</th>
<th>Spacing</th>
<th>Rspeed</th>
<th>AngDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>Alternative 1</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>10.38</td>
<td>-0.08</td>
<td>4.80</td>
</tr>
<tr>
<td></td>
<td>Alternative 2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>20.53</td>
<td>0.94</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>Alternative 3</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>16.58</td>
<td>-0.02</td>
<td>0.66</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>Alternative 1</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>9.43</td>
<td>-1.50</td>
<td>5.50</td>
</tr>
<tr>
<td></td>
<td>Alternative 2</td>
<td>2</td>
<td>5</td>
<td>0</td>
<td>15.47</td>
<td>-1.66</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>Alternative 3</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>20.53</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>Alternative 4</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>25.64</td>
<td>0.25</td>
<td>1.87</td>
</tr>
<tr>
<td></td>
<td>Alternative 5</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>8.81</td>
<td>-1.04</td>
<td>7.79</td>
</tr>
</tbody>
</table>

5 Model calibration and validation procedure

The proposed two steps of the modelling have distinctive nature, where the estimation problem at the first step (area selection) is discrete, however, the problem at the second step (vehicle following) is continuous. Integrating these two steps and estimating their parameters jointly should provide optimal parameter results. In literature, researchers (Toledo, Koutsopoulos, and Ben-Akiva (2009), Koutsopoulos and Farah (2012) and Choudhury and Islam (2016)) have adopted the joint calibration process. Here, the probability density function for the continuous model need to be derived by assuming a distribution for its error component. The objective of this paper is to develop a modelling framework for heterogeneous traffic and demonstrate its satisfying performance, achieving the optimality of the parameter estimation is not critical. Therefore, in this research we adopt a separate calibration method which is simple and easy to implement. The separate calibration details for the step-1 and step-2 are presented in section 6 and 7, respectively. The calibrated parameters are more conservative compared with the would-be parameters from the simultaneous calibration method, in the sense that using the simultaneous calibration method would only make our model performance better.

Moreover, to fully capture the whole model’s performance we not only separately assessed each step’s performance but also sequentially assess the whole model’s performance (section 8), where the error from the first step is propagated to the second step.

Note:

a) For area selection model calibration, the MNL model is calibrated for cars (section 6.3) and motorcycles (section 6.5) independently using 70% of the data.

b) For area selection model validation, the MNL model for cars (section 6.4) and motorcycles (section 6.6) is validated using the remaining 30% of the data.

6 Step-1: Area selection model development, calibration and validation

This section is structured as follows: First, the performance indicators used for the calibration of area selection step is presented in section 6.1, followed by the details of the three potential
MNL models for area selection in section 6.2. Thereafter, the MNL model selection for car, its calibration results and independent validation is presented in sections 6.2, 6.3 and 6.4, respectively. The same process of motorcycle is presented in sections 6.2, 6.5 and 6.6, respectively.

### 6.1 Performance matrices for calibration of MNL model

The performance matrices during the calibration of the MNL model is quantified using the following indicators:

**True positive (TP):** The actual and predicted positions are same in any specific alternative.

**False positive (FP):** The actual position is not in a specific alternative, but the predicted position is in a specific alternative.

**False negative (FN):** The actual position is in a specific alternative, but the predicted position is not in a specific alternative.

**True negative (TN):** The actual and predicted positions are not in a specific alternative.

**Sensitivity:** The term “sensitivity” (Lantz (2013)) measures the ability of a prediction model to correctly predict the position in a specific alternative when the actual position is in that alternative. Thus, sensitivity is defined as:

$$ Sensitivity = \frac{TP}{TP + FN} \tag{10} $$

where $TP$ is the number of true positives and $FN$ is the number of false negatives.

**Specificity:** The term “specificity” (Lantz (2013)) is concerned with how good the prediction model is at correctly identifying alternatives, and is defined as:

$$ Specificity = \frac{TN}{TN + FP} \tag{11} $$

where $TN$ is the number of true negatives and $FP$ is the number of false positives.

A high sensitivity alone is not necessarily a good indicator for the prediction model. A naïve model that always predicts the specific alternative will have a sensitivity of 100%, but a specificity of 0%. A perfect prediction (i.e., all predictions are correct) has both sensitivity and specificity of 100%. Therefore, both sensitivity and specificity should be considered while interpreting the results.

**Positive Predictive Value:** The positive predictive value (PPV) is the proportion of the positive values that are accurately predicted; that is, the ratio of $TP$ to the total number of positive values ($TP + FP$), which can be defined as:

$$ PPV = \frac{TP}{TP + FP} \tag{12} $$

where $FP$ is the number of false positives.

**Negative Predictive Value:** The negative predictive value (NPV) is the proportion of negative values that are accurately predicted; that is, the ratio of $TN$ to the total number of negative values ($TN + FN$), which can be defined as:

$$ NPV = \frac{TN}{TN + FN} \tag{13} $$
where $FN$ is the number of false negatives.

The significance of the positive and negative predictive values is that they change if the number of actual observations for a specific alternative change. Indeed, for any prediction model, the positive predictive value will fall as the number of actual observations for a specific alternative fall, while the negative predictive value will rise.

**Accuracy**: The accuracy label for prediction of each alternative can be measured by the following equation:

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$  \hspace{1cm} (14)

**Receiver operating characteristics**: Receiver operating characteristics (ROC) graphs have become a useful tool for analysing and visualising the performance of the prediction model in various fields, such as medical decision making, machine learning, and data mining research (Everson and Fieldsend (2006)). ROC space is defined by a two-dimensional graph in which the X-axis represents the false positive rate (i.e., 1 - specificity) and the Y-axis represents the true positive rate (i.e., sensitivity). The diagonal line in the ROC space represents the strategy of randomly guessing an alternative. Any point for the prediction of each alternative from a model that appears below the diagonal line performs worse than random guessing. The upper left corner of the ROC space represents 100% sensitive and 100% specificity. The closer the ROC point is to the upper left corner, the higher the overall accuracy.

### 6.2 Area selection model development

Equation (4) provides the generic equation for the MNL. For the modelling, the alternative specific constant ($\lambda$), for alternative 3 for car ($\lambda_3$) and alternative 5 for motorcycle ($\lambda_5$) is assumed to be zero. In addition, following three MNL models are build and their performance is compared:

**Model-1**: Here, all the modelling parameters are considered. It has 11 parameters for car and 19 for motorcycles.

**Model-2**: In this model, the utility of any alternative is assumed to not rely on the unobserved effects of such alternative. Specifically, to test this hypothesis, all alternative specific constants are assumed to be zero; that is, $\lambda_n = 0$ in Equation (4). It has 9 parameters for car and 15 parameters for motorcycle.

**Model-3**: In this model, the coefficients of the same attributes are considered generic. That is, $\beta_{n,1} = \beta_{n,2} = \beta_{n,3} = \beta_n$, resulting in 5 parameters for car and 7 parameters for motorcycle.

Table 2 shows the specific number of parameters to be estimated for each type of vehicle based on the number of alternatives in utility and different model selection.

The log-likelihood ratio test is considered to compare these three models with different levels of complexity. The test is performed using Equation (15):

$$LR = -2(LL_R - LL_U)$$  \hspace{1cm} (15)

where $LR$ represents the log-likelihood ratio, $LL_R$ represents the log-likelihood value for the MNL model with restricted parameters (i.e., the number of parameters is 9 and 5 for cars, 15 and 7 for motorcycles in Model-2 and Model-3, respectively) and $LL_U$ represents the log-likelihood value for the MNL model with unrestricted parameters (i.e., number of parameters is 11 for cars, 19 for motorcycles in Model-1).
Table 2 Number of parameters to be estimated in different models

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>No. of alternatives in utility</th>
<th>No. of parameters to be estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Model-1</td>
</tr>
<tr>
<td>Car</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

Table 3 presents the log-likelihood values of the three MNL models for cars. The LR values for model-2 and model-3 was significantly higher than the respective critical value at the 95th percentile in the Chi square distribution for 2 degree and 6 degree at the 5% level, respectively. Therefore, these models were rejected and Model-1 is selected for car.

Table 3 Different log-likelihood values of MNL models for car

<table>
<thead>
<tr>
<th>MNL models</th>
<th>No. of parameter</th>
<th>No. of observation</th>
<th>Log-likelihood value</th>
<th>LR value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-1</td>
<td>11</td>
<td>17754</td>
<td>-10986.40</td>
<td></td>
</tr>
<tr>
<td>Model-2</td>
<td>9</td>
<td>17754</td>
<td>-11270.11</td>
<td>567.42*</td>
</tr>
<tr>
<td>Model-3</td>
<td>5</td>
<td>17754</td>
<td>-14025.48</td>
<td>6078.16**</td>
</tr>
</tbody>
</table>

* LR value was significantly higher than the critical value (5.99) at the 95th percentile in the Chi-square distribution of 2 degrees at the 5% level. Model-2 was rejected.

**LR value was significantly higher than the critical value (12.59) at the 95th percentile in the Chi-square distribution of 6 degrees at the 5% level. Model-3 was rejected.

Similarly, Table 4 presents the log-likelihood values of the three MNL models for motorcycle. The LR values for Model-2 and Model-3 was significantly higher than the respective critical value at the 95th percentile in the Chi square distribution for 4 degrees and 12 degrees at the 5% level, respectively. Therefore, these models were rejected and Model-1 is selected for motorcycles.

Table 4 Different log-likelihood values of the MNL models for vehicle type “Motorcycle”

<table>
<thead>
<tr>
<th>MNL models</th>
<th>No. of parameter</th>
<th>No. of observation</th>
<th>Log-likelihood value</th>
<th>LR value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-1</td>
<td>19</td>
<td>37941</td>
<td>-18947.53</td>
<td></td>
</tr>
<tr>
<td>Model-2</td>
<td>15</td>
<td>37941</td>
<td>-19571.85</td>
<td>1248.64*</td>
</tr>
<tr>
<td>Model-3</td>
<td>7</td>
<td>37941</td>
<td>-19504.12</td>
<td>1113.18**</td>
</tr>
</tbody>
</table>

* LR value was significantly higher than the critical value (9.49) at the 95th percentile in the Chi-square distribution of 4 degrees at the 5% level. Model-2 was rejected.

**LR value was significantly higher than the critical value (21.03) at the 95th percentile in the Chi-square distribution of 12 degrees at the 5% level. Model-3 was rejected.

6.3 MNL model calibration for car

The total number of observations in the training dataset for cars is 17,754. The calibrated values using maximum likelihood estimation of all of the coefficients of attributes and ASCs for Equation (4) are listed in Table 5. The estimated parameters of the selected MNL model for cars are significant. The estimated coefficients for spacing ($\beta_{n1}$) and relative speed ($\beta_{n2}$)
where $n = 1, 2, 3$ in all alternatives are positive. This means that the utility of an alternative increased with an increase in spacing or relative speed. The coefficients for angle deviation ($\beta_{n,3}$) where $n = 1, 2, 3$ in all alternatives have a negative sign. The utility of an alternative decreased with an increase in angular deviation. The subject vehicle tends to prefer nearby alternatives.

Table 6 represents the summarising number of TP, FP, TN, and FN for the centre, left, and right alternatives, respectively. Table 7 represents the performance of MNL model during calibration. In general, the accuracy of the model during calibration process is more than 84%. Due to higher proportion of FN (over TP) for right and left alternatives, the sensitivity for these alternatives is lower than that of centre alternative. Due to high proportion of FP (over TN) for centre alternative, the specificity for centre alternative is lower than that of right and left alternatives. Nevertheless, the ROC data points during calibration process (refer to calibration points in Figure 8) are above the diagonal line, representing acceptable overall accuracy from the calibration process.

Table 5 Estimated parameters of the selected MNL model for cars

<table>
<thead>
<tr>
<th>Name of Coefficient</th>
<th>Coefficient</th>
<th>Estimated Value</th>
<th>Standard error</th>
<th>95% Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASC for Centre</td>
<td>$\lambda_2$</td>
<td>3.128</td>
<td>0.140</td>
<td>2.854 - 3.401</td>
</tr>
<tr>
<td>ASC for Left</td>
<td>$\lambda_3$</td>
<td>0.288</td>
<td>0.110</td>
<td>0.072 - 0.504</td>
</tr>
<tr>
<td>Spacing for Right</td>
<td>$\beta_{1,1}$</td>
<td>0.022</td>
<td>0.004</td>
<td>0.014 - 0.029</td>
</tr>
<tr>
<td>Relative Speed for Right</td>
<td>$\beta_{1,2}$</td>
<td>0.070</td>
<td>0.017</td>
<td>0.037 - 0.103</td>
</tr>
<tr>
<td>Angle Deviation for Right</td>
<td>$\beta_{1,3}$</td>
<td>-0.272</td>
<td>0.014</td>
<td>-0.298 - -0.245</td>
</tr>
<tr>
<td>Spacing for Centre</td>
<td>$\beta_{2,1}$</td>
<td>0.023</td>
<td>0.005</td>
<td>0.012 - 0.033</td>
</tr>
<tr>
<td>Relative Speed for Centre</td>
<td>$\beta_{2,2}$</td>
<td>0.290</td>
<td>0.019</td>
<td>0.253 - 0.328</td>
</tr>
<tr>
<td>Angle Deviation for Centre</td>
<td>$\beta_{2,3}$</td>
<td>-3.456</td>
<td>0.055</td>
<td>-3.564 - -3.348</td>
</tr>
<tr>
<td>Spacing for Left</td>
<td>$\beta_{3,1}$</td>
<td>0.039</td>
<td>0.004</td>
<td>0.032 - 0.047</td>
</tr>
<tr>
<td>Relative Speed for Left</td>
<td>$\beta_{3,2}$</td>
<td>0.394</td>
<td>0.018</td>
<td>0.357 - 0.428</td>
</tr>
<tr>
<td>Angle Deviation for Left</td>
<td>$\beta_{3,3}$</td>
<td>-0.475</td>
<td>0.013</td>
<td>-0.501 - -0.449</td>
</tr>
</tbody>
</table>

Table 6 Illustration of TP, FP, TN and FN for each alternative during model calibration on car

<table>
<thead>
<tr>
<th>Actual position in alternative</th>
<th>Right</th>
<th>Not Right</th>
<th>Centre</th>
<th>Not Centre</th>
<th>Left</th>
<th>Not Left</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted position in alternative</td>
<td>Right</td>
<td>1192</td>
<td>601</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Not Right</td>
<td>1897</td>
<td>14064</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Centre</td>
<td>-</td>
<td>-</td>
<td>10504</td>
<td>2413</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Not Centre</td>
<td>-</td>
<td>-</td>
<td>411</td>
<td>4426</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Left</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2015</td>
</tr>
<tr>
<td></td>
<td>Not Left</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1735</td>
</tr>
</tbody>
</table>
Table 7 MNL model performance measure during calibration for car

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Dataset</th>
<th>MNL model performance measured during calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sensitivity (%)</td>
</tr>
<tr>
<td>Right</td>
<td>Training</td>
<td>38.59</td>
</tr>
<tr>
<td>Centre</td>
<td>Training</td>
<td>96.23</td>
</tr>
<tr>
<td>Left</td>
<td>Training</td>
<td>53.73</td>
</tr>
</tbody>
</table>

6.4 MNL model validation for car

The performance of the MNL model for cars is measured using the validation dataset (i.e., testing dataset). The number of observations for each alternative in the training and testing datasets is presented in Figure 7. The total number of observations is 17,754 in the training data, whereas it is 7,113 in the testing data.

Considering centre as the prediction alternative, the ratio of centre to not centre data points is approximately 61:39 in the training dataset and 75:25 in the testing dataset. Similarly, for left as the prediction alternative, the ratio of left to not left data points is 21:79 in the training dataset and 15:85 in the testing dataset. In addition, for right as the prediction alternative, the ratio of right to not right data points is 17:83 in the training dataset and 10:90 in the testing dataset.

The aggregated proportion of correct predictions was measured by summing across the number of correct predictions and dividing by the total number of choices made. For the training data (where model parameters are estimated) it was 79.93% and the testing data was 79.33%. The absolute difference of the aggregated proportion of the correct prediction between these two datasets was 0.6%, which ensured the consistency of correct prediction using the MNL model.

Table 8 summarises the model performance for each alternative selection in terms of sensitivity, specificity, PPV, NPV, and accuracy. The significant variations of these indicators
are observed in validation dataset. These indicators could be sensitive due to different ratios of alternatives in observed data and lower sample size for each alternative.

Figure 8 presents the ROC graph for MNL model for calibration and validation dataset. The prediction for “centre” is significantly different than others due to observed significant variations in sample size from other alternatives. Nevertheless, the points from the alternatives in both cases are in the upper half of the diagonal line, which indicates a reasonable performance from the model.

Table 8 The performance of MNL model on validation dataset for car

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Dataset</th>
<th>Sensitivity (%)</th>
<th>Specificity (%)</th>
<th>PPV (%)</th>
<th>NPV (%)</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right</td>
<td>Testing</td>
<td>70.01</td>
<td>89</td>
<td>43.12</td>
<td>96.14</td>
<td>86.98</td>
</tr>
<tr>
<td>Centre</td>
<td>Testing</td>
<td>83.82</td>
<td>90.58</td>
<td>96.29</td>
<td>65.75</td>
<td>85.55</td>
</tr>
<tr>
<td>Left</td>
<td>Testing</td>
<td>63.55</td>
<td>90.09</td>
<td>52.87</td>
<td>93.39</td>
<td>86.14</td>
</tr>
</tbody>
</table>

Figure 8 A ROC graph illustrates the performance of the MNL model for car in different time steps during calibration and validation process.

6.5 MNL model calibration for motorcycle

The estimated parameters of the selected MNL model for motorcycles are significant and are listed in Table 9. The estimated coefficients for spacing ($\beta_{n,1}$) and relative speed ($\beta_{n,2}$) for all alternatives are positive. The coefficients for angle deviation ($\beta_{n,3}$) for all alternatives are negative. This is consistent with that of car.

Table 10 summarizes the number of TP, FP, TN, and FNs for all alternatives. The summary of the prediction measured for the MNL model for motorcycles during calibration is presented in Table 11. The sensitivity for alternative 1 and alternative 5 is very low due to lower sample size of these two alternatives in training dataset. The prediction accuracy for alternative 1 and alternative 5 is almost the same (i.e., approximately 97%) and for alternative 2 and alternative
4 is approximately 90%, whereas the prediction accuracy is slightly lower for alternative 3 (approximately 87%). The corresponding ROC graph (calibration data points in Figure 9) data points for all the alternatives above the diagonal line, which indicates a reasonable performance from the model during calibration.

Table 9 Estimated parameters of the selected MNL model for motorcycles

<table>
<thead>
<tr>
<th>Name of Coefficient</th>
<th>Coefficient</th>
<th>Estimated Value</th>
<th>Standard error</th>
<th>95% Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASC for Alternative 1</td>
<td>( \lambda_1 )</td>
<td>1.189</td>
<td>0.189</td>
<td>1.126 1.867</td>
</tr>
<tr>
<td>ASC for Alternative 2</td>
<td>( \lambda_2 )</td>
<td>1.877</td>
<td>0.180</td>
<td>1.584 2.288</td>
</tr>
<tr>
<td>ASC for Alternative 3</td>
<td>( \lambda_3 )</td>
<td>4.849</td>
<td>0.176</td>
<td>4.205 4.893</td>
</tr>
<tr>
<td>ASC for Alternative 4</td>
<td>( \lambda_4 )</td>
<td>1.748</td>
<td>0.183</td>
<td>1.288 2.005</td>
</tr>
<tr>
<td>Spacing for Alternative 1</td>
<td>( \beta_{1,1} )</td>
<td>0.037</td>
<td>0.005</td>
<td>0.014 0.033</td>
</tr>
<tr>
<td>Relative Speed for Alternative 1</td>
<td>( \beta_{1,2} )</td>
<td>0.702</td>
<td>0.026</td>
<td>0.570 0.671</td>
</tr>
<tr>
<td>Angle Deviation for Alternative 1</td>
<td>( \beta_{1,3} )</td>
<td>-1.689</td>
<td>0.061</td>
<td>-1.868 -1.628</td>
</tr>
<tr>
<td>Spacing for Alternative 2</td>
<td>( \beta_{2,1} )</td>
<td>0.107</td>
<td>0.004</td>
<td>0.099 0.114</td>
</tr>
<tr>
<td>Relative Speed for Alternative 2</td>
<td>( \beta_{2,2} )</td>
<td>0.470</td>
<td>0.014</td>
<td>0.397 0.454</td>
</tr>
<tr>
<td>Angle Deviation for Alternative 2</td>
<td>( \beta_{2,3} )</td>
<td>-2.711</td>
<td>0.045</td>
<td>-2.916 -2.741</td>
</tr>
<tr>
<td>Spacing for Alternative 3</td>
<td>( \beta_{3,1} )</td>
<td>0.027</td>
<td>0.004</td>
<td>0.023 0.037</td>
</tr>
<tr>
<td>Relative Speed for Alternative 3</td>
<td>( \beta_{3,2} )</td>
<td>0.495</td>
<td>0.013</td>
<td>0.424 0.475</td>
</tr>
<tr>
<td>Angle Deviation for Alternative 3</td>
<td>( \beta_{3,3} )</td>
<td>-3.397</td>
<td>0.042</td>
<td>-3.298 -3.134</td>
</tr>
<tr>
<td>Spacing for Alternative 4</td>
<td>( \beta_{4,1} )</td>
<td>0.097</td>
<td>0.004</td>
<td>0.90 0.105</td>
</tr>
<tr>
<td>Relative Speed for Alternative 4</td>
<td>( \beta_{4,2} )</td>
<td>0.583</td>
<td>0.016</td>
<td>0.507 0.570</td>
</tr>
<tr>
<td>Angle Deviation for Alternative 4</td>
<td>( \beta_{4,3} )</td>
<td>-2.644</td>
<td>0.0447</td>
<td>-2.737 -2.562</td>
</tr>
<tr>
<td>Spacing for Alternative 5</td>
<td>( \beta_{5,1} )</td>
<td>0.074</td>
<td>0.007</td>
<td>0.063 0.089</td>
</tr>
<tr>
<td>Relative Speed for Alternative 5</td>
<td>( \beta_{5,2} )</td>
<td>0.606</td>
<td>0.025</td>
<td>0.569 0.668</td>
</tr>
<tr>
<td>Angle Deviation for Alternative 5</td>
<td>( \beta_{5,3} )</td>
<td>-1.675</td>
<td>0.061</td>
<td>-1.841 -1.602</td>
</tr>
</tbody>
</table>

Table 10 Illustration of TP, FP, TN and FN for each alternative during model calibration

<table>
<thead>
<tr>
<th>Actual position in alternative</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
<th>Alternative 3</th>
<th>Alternative 4</th>
<th>Alternative 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted position in alternative</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternative 1</td>
<td>376</td>
<td>60</td>
<td>42</td>
<td>34</td>
<td>30</td>
</tr>
<tr>
<td>Alternative 2</td>
<td>287</td>
<td>6571</td>
<td>837</td>
<td>443</td>
<td>172</td>
</tr>
<tr>
<td>Alternative 3</td>
<td>241</td>
<td>1389</td>
<td>18663</td>
<td>1411</td>
<td>284</td>
</tr>
<tr>
<td>Alternative 4</td>
<td>133</td>
<td>459</td>
<td>659</td>
<td>4939</td>
<td>289</td>
</tr>
<tr>
<td>Alternative 5</td>
<td>28</td>
<td>48</td>
<td>58</td>
<td>70</td>
<td>418</td>
</tr>
</tbody>
</table>
Table 11 Summary of the performance measured of the MNL model for motorcycles during calibration

<table>
<thead>
<tr>
<th>Name of alternatives</th>
<th>MNL model performance measured on calibration data for motorbikes</th>
<th>Sensitivity (%)</th>
<th>Specificity (%)</th>
<th>PPV (%)</th>
<th>NPV (%)</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 1</td>
<td></td>
<td>35.31</td>
<td>99.55</td>
<td>69.37</td>
<td>98.16</td>
<td>97.75</td>
</tr>
<tr>
<td>Alternative 2</td>
<td></td>
<td>77.06</td>
<td>94.09</td>
<td>79.07</td>
<td>93.40</td>
<td>90.26</td>
</tr>
<tr>
<td>Alternative 3</td>
<td></td>
<td>92.12</td>
<td>81.20</td>
<td>84.88</td>
<td>90.0</td>
<td>87.03</td>
</tr>
<tr>
<td>Alternative 4</td>
<td></td>
<td>71.61</td>
<td>95.04</td>
<td>76.23</td>
<td>93.78</td>
<td>90.78</td>
</tr>
<tr>
<td>Alternative 5</td>
<td></td>
<td>35.04</td>
<td>99.44</td>
<td>67.20</td>
<td>97.92</td>
<td>97.42</td>
</tr>
</tbody>
</table>

6.6 MNL model validation for motorcycle

The performance of the MNL model for motorcycle is also measured on the validation dataset (i.e., testing dataset). The total number of observations is 37,941 in the training data, whereas it was 14,353 in the testing data. For training data, the aggregated proportion of correct prediction is 91.37% and in the testing data it is 83.59%. Table 12 summarises the model performance in terms of sensitivity, specificity, PPV, NPV, and accuracy. The ROC graph in Figure 9 illustrates the performance of the MNL model for motorcycles during calibration. The plotted points from the alternatives are in the upper half of the diagonal line, which indicates a reasonable performance from the model.

Table 12 The performance of MNL model for motorcycle for the validation data

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Dataset</th>
<th>MNL model performance measured for each alternative</th>
<th>Sensitivity (%)</th>
<th>Specificity (%)</th>
<th>PPV (%)</th>
<th>NPV (%)</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 1</td>
<td>Testing</td>
<td></td>
<td>39.21</td>
<td>99.63</td>
<td>74.13</td>
<td>98.37</td>
<td>98.03</td>
</tr>
<tr>
<td>Alternative 2</td>
<td>Testing</td>
<td></td>
<td>79.27</td>
<td>94.71</td>
<td>80.03</td>
<td>94.47</td>
<td>91.45</td>
</tr>
<tr>
<td>Alternative 3</td>
<td>Testing</td>
<td></td>
<td>93.37</td>
<td>83.41</td>
<td>87.09</td>
<td>91.30</td>
<td>88.84</td>
</tr>
<tr>
<td>Alternative 4</td>
<td>Testing</td>
<td></td>
<td>73.89</td>
<td>95.27</td>
<td>78.38</td>
<td>94.02</td>
<td>91.24</td>
</tr>
<tr>
<td>Alternative 5</td>
<td>Testing</td>
<td></td>
<td>34.86</td>
<td>99.50</td>
<td>67.44</td>
<td>98.08</td>
<td>97.62</td>
</tr>
</tbody>
</table>
A ROC graph illustrates the performance of the MNL model for motorcycle during calibration and validation process.

7 Stage-2: Vehicle following model calibration

MIDM parameters are independently calibrated for each subject vehicle. In contrast to the traditional CF model calibration where the leader and follower (subject vehicle) pair is the same; here, the leader of the subject vehicle often changes at different simulation time steps. The alternatives defined in first step of this modelling are considered consistent in the second step of this modelling for vehicle movement model calibration. At each time step, the movement of the subject vehicle is defined along the direction of the alternative. The leader is the vehicle (within the alternatives) with minimum spacing with that of the subject vehicle. The vector projection of the spacing and relative speed of a subject vehicle along the direction of an alternative defines the respective parameters required to calibrate MIDM. The error between the direction of the movement of the subject vehicle along the direction of the alternative and the actual direction of the movement of the subject vehicle can result in a calibration error.

MIDM model parameters are calibrated by minimizing the RMSE (Equation (16)) between the estimated and actual vehicle trajectory. Genetic algorithm (GA) is used to solve this optimisation problem.

\[ RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_i^{sim} - X_i^{data})^2 + (Y_i^{sim} - Y_i^{data})^2} \]  

(16)

where, \((X_i^{sim}, Y_i^{sim})\) and \((X_i^{data}, Y_i^{data})\) are simulated and actual (ground truth) position of a subject vehicle, respectively.

The model is calibrated based on the following bounds of each parameter:
\[ 10 \leq v_d (km/h) \leq 110 \]
\[ 0 < a_{max} (m/s^2), b (m/s^2), T (s), s_0 (m), s_1 (m) \leq 10 \]
\[ \delta = 4 \]

7.1 MIDM calibration results for car

The empirical distribution of the calibrated parameter for 130 cars is presented in Figure 10. The mean for desired speed, maximum acceleration, desired deceleration, safety time headway, linear jam distance, and non-linear jam distance are 67.78 (km/h), 3.31 (m/s^2), 2.44 (m/s^2), 0.74 (s), 1.65 (m), and 1.99 (m), respectively. The standard deviations are 22 (km/h), 1.37 (m/s^2), 1.44 (m/s^2), 0.36 (s), 1.20 (m), and 1.41 (m) for desired speed, maximum acceleration, desired deceleration, safety time headway, linear jam distance, and non-linear jam distance, respectively. The distributions of the RMSE values for the calibrated trajectories of cars are shown in Figure 11. The mean and standard deviation of the RMSE were measured as 4.26 (m) and 1.64 (m), respectively.

Figure 10 Distribution of calibrated parameters of the MIDM from 130 car trajectories.

Figure 11 Distribution of RMSE of the MIDM from 130 car trajectories.
7.2 MIDM calibration results for motorcycle

The distribution of the calibrated parameters for 40 motorcycles is presented in Figure 12. The distribution shows the frequency of the estimated values of the parameters. The means for desired speed, maximum acceleration, desired deceleration, safety time headway, linear jam distance, and non-linear jam distance are 43.94 (km/h), 5.91 (m/s²), 2.96 (m/s²), 0.29 (s), 1.40 (m), and 1.60 (m), respectively. The standard deviations are 9.39 (km/h), 2.08 (m/s²), 2.96 (m/s²), 0.24 (s), 1.30 (m), and 2.11 (m) for desired speed, maximum acceleration, desired deceleration, safety time headway, linear jam distance, and non-linear jam distance, respectively. The distributions of the RMSE values for the calibrated trajectories of motorcycles are shown in Figure 13. The mean and standard deviation of RMSE are measured as 3.16 (m) and 1.84 (m), respectively.

![Figure 12 Distribution of calibrated parameters of the MIDM from 40 motorcycle trajectories.](image1)

![Figure 13 Distribution of RMSE of the MIDM from 40 motorcycle trajectories.](image2)
8 Entire model verification: Simulating area-based traffic

Here, the entire modelling is applied to simulate the vehicle trajectories using the initial and boundary conditions defined from the aforementioned real data (Section 4). The simulation section is from 50 m to 150 m of the road length. The initial condition is defined for the entire road at time 52.5 seconds. The boundary conditions are defined using the observed data from first 50 m and from 150m to 245 m. Specifically,

- **Initial condition**: Observed vehicle positions and dynamics (speed, acceleration) at time 52.5 seconds for the entire road from 0m to 245m is the initial condition.
- **Upstream boundary condition**: The observed vehicle positions and dynamics (speed, acceleration) within the 0m to 50m defines the upstream boundary condition. In other words, for simulation the vehicles are generated as per their actual origin in the real data within the first 50m of the study section.
- **Downstream boundary condition**: The observed actual average vehicle speed in the space time region from 150m to 245m at each time interval of 60s defines the downstream boundary condition.

As presented in section 6 and 7, the model parameters are calibrated for cars and motorcycles only. The real data has low percentage of auto-rickshaws (12.2%) and heavy vehicles (4.8%), due to which it is not possible to have a satisfactory independent calibration for these vehicle types. Therefore, for the current application these vehicles are treated as cars. This is a conservative approach to test the entire model performance in simulating the vehicle trajectories.

The simulation is performed for five replications. The parameters from the second step (MIDM) are stochastically assigned for each replication using the distribution presented in section 7. However, the parameters from first step (area selection) are deterministic for each vehicle type and remained constant for all replications.

The simulated trajectories are macroscopically compared with that from the real observations using Edie's (1963) generalised definition of the density and flow in space-time region is considered, where the total time taken (TTT) and total distance travelled (TDT) by each vehicle in the time space region is the proxy for density and flow, respectively.

\[
density = \frac{\text{TTT}}{\text{Area for time space region}} = \frac{\sum_{i=1}^{n} r_i}{xt} \quad (17)
\]

\[
flow = \frac{\text{TDT}}{\text{Area for time space region}} = \frac{\sum_{i=1}^{n} d_i}{xt} \quad (18)
\]

where \( r_i \) represents the time taken for a particular vehicle \( i \) over defined rectangular region; \( d_i \) represents the distance travelled for the vehicle \( i \) over that rectangular region; and \( n \) is the number of vehicle trajectories in the region.

For the current application the time space-region for the simulated and real trajectories are divided into windows of 60 seconds (along time axis) and 50 m to 150 m (along space axis), i.e., \( t = 60(s) \) and \( x = 100(m) \). The simulation is performed from 52.5 seconds to 1792.5 seconds, which leads to 29 such time space windows. Figure 14, illustrates the actual (observed) trajectories for one such time space region. The corresponding simulated trajectory for one of the replications is presented in Figure 15.
Figure 14 Observed vehicle trajectories in a 60 (s) x 100 (m) time-space region.

Figure 15 Simulated vehicle trajectories in a 60 (s) x 100 (m) time-space region.
The results from the five replications over 29 of such regions is presented in Figure 16, Figure 17, and Figure 18. Here

a) Figure 16 presents the number of simulated vehicle trajectories in each time-space region. The number of simulated vehicle trajectories are slightly deviated from the observed number of vehicle trajectories in that region.

b) Figure 17 presents the results for the total time taken in each time space region. Theil inequality coefficient and R-square of the statistical performance measures are used to measure the performance of the entire model. The Theil inequality coefficient is bounded by 0 and 1. The lower boundary of this inequality is the case of ideal forecast of the model while the upper boundary stands for the prediction model is naïve or trivial. The Theil inequality coefficient is approximately 0.02 for each replication and the R-squared value is 0.92.

c) Figure 18 presents the results for the total distant travelled in each time space region. The Theil inequality coefficient is approximately 0.01 for each replication and the R-squared value is 0.95.

The above results indicate that the overall model performance is satisfactory, and the trajectories can be reasonably simulated using the proposed modelling framework for heterogeneous traffic conditions.
Figure 17 a) Illustration of time series of the total time taken (TTT in seconds) for vehicles defined in time-space windows; and b) The comparison of the observed (x-axis) and simulated (y-axis) TTT using 45-degree line.
Figure 18 a) Illustration of time series of the total distance travelled (TDT in meters) for vehicles defined in the time-space windows; and b) The comparison of the observed (x-axis) and simulated (y-axis) TDT using 45-degree line.

![Graph a)](image)

### Graph a)
- **Observed (ground truth)**
- Simulation replication 1
- Simulation replication 2
- Simulation replication 3
- Simulation replication 4
- Simulation replication 5

### Graph b)
- Reference line
- Simulation replication 1
- Simulation replication 2
- Simulation replication 3
- Simulation replication 4
- Simulation replication 5

**Legend:**
- **Observed TDT (meters)**
- **Simulated TDT (meters)**

**Axes:**
- **Time Interval in Seconds**
- **Total Distance Taken (TDT in meters)**
9 Conclusions and future research

This paper proposed a microscopic model to simulate the vehicle movement in area-based heterogeneous traffic. The model has two steps: area selection and vehicle movement.

1. Area selection: This step identifies the direction of movement of the vehicle. For this a discrete choice modelling framework is considered. The potential movement directions are considered as alternatives, the attribute for which are defined in terms of angular deviation from the direction of the flow, spacing and relative speed. The selection process is modelled using multinomial logit model (MNL). This step is developed and validated for both cars and motorcycles. The direction of an alternative for the subject vehicle as a car is correctly captured approximately 84% to 86% for each alternative prediction. The direction of alternative for subject vehicle as a motorcycle is captured approximately 87% to 97% for each alternative predication. The performance of MNL model during calibration and on validation dataset are also measured using ROC graphs which supports the applicability of this model.

2. Vehicle movement: For vehicle movement a modified version of IDM model, termed as MIDM is developed. The parameters of MIDM are independently calibrated for cars and motorcycles. The empirical distributions of the calibrated parameters for cars and motorcycles is later utilized for stochastic simulation.

The applicability of the entire modelling framework to simulate traffic is verified through its application on the real dataset, where the real data provide the initial and boundary conditions for simulation. The performance is evaluated at macroscopic level through the estimation of total distance travelled (TDT) and total time taken (TTT) by vehicle trajectories within space-time windows. Comparing the results from simulation and real dataset, it is observed that the Theil inequality coefficient for TTT is approximately 0.02 for each simulation replications and the R-squared value is 0.92. Similarly, the Theil inequality coefficient for TDT is approximately 0.01 for each replication and the R-squared value is 0.95. These results indicate that the overall model performance is satisfactory, and the trajectories can be reasonably simulated using the proposed modelling framework for heterogeneous traffic conditions. The performance can be further enhanced by consider a) joint calibration of the model parameters and b) considering of driver heterogeneity. Such enhancement in the model performance is at the cost of additional complexity in the modelling and can be a scope of future research.

The objective for the current research is to propose the methodology and test is performance at satisfactory level. It is proposed to extend the research in the following directions:

a) Extend the attributes of the area selection step through consideration of additional human factors related to perfection error and delay.

b) Currently the modelling has two steps as hierarchy. It is proposed to explore other formulations where the hierarchical decision is considered at different tactical and operations levels, as reviewed by Michon (1985).

c) Extending the modelling through different frequency of update for the area selection.

d) Extend the methodology to incorporate different vehicles, especially during merging and diverging section. Further this, datasets from such facilities needs to be collected.

e) The model calibration process can be enhanced considering the joint calibration as proposed by Toledo et al. (2009), Koutsopoulos and Farah (2012) and Choudhury and Islam (2016).
f) For the current approach heterogeneity amongst the drivers has been ignored. As a future research, it is recommended to consider mixed logit model where *spacing, relative speed, and angle deviation* can be treated as random parameters.

**Acknowledgment**

The authors are grateful to Kanagaraj et al. (2015) for sharing the trajectory data used in this research. The financial support from the Queensland University of Technology is acknowledged.

**Appendix A**

**Estimated parameters of the MNP model for car**

<table>
<thead>
<tr>
<th>Choice Set</th>
<th>Right</th>
<th>Centre</th>
<th>Left</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(n = 1)$</td>
<td>$(n = 2)$</td>
<td>$(n = 3)$</td>
</tr>
<tr>
<td>Constant $\lambda_n$, $n = 1,2,3$</td>
<td>-4.263</td>
<td>-4.053</td>
<td>0.000</td>
</tr>
<tr>
<td>Spacing</td>
<td>0.019</td>
<td>0.033</td>
<td>0.038</td>
</tr>
<tr>
<td>Coefficient matrix $\beta_{i,j}$, $i,j = 1,2,3$</td>
<td>RSpeed: 0.081</td>
<td>0.384</td>
<td>0.371</td>
</tr>
<tr>
<td></td>
<td>AngDev: -0.278</td>
<td>-5.067</td>
<td>-0.465</td>
</tr>
<tr>
<td>Standard deviations</td>
<td>1.000</td>
<td>2.629</td>
<td>1.000</td>
</tr>
<tr>
<td>Correlations</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Centre</td>
<td>0.000</td>
<td>1.000</td>
<td>-0.006</td>
</tr>
<tr>
<td>Left</td>
<td>0.000</td>
<td>-0.006</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**References**


Lee, T.-C., Polak, J., & Bell, M. (2009). New approach to modeling mixed traffic containing motorcycles in urban areas. Transportation Research Record: Journal of the Transportation Research Board(2140), 195-205.


