On the periodicity of traffic oscillations and capacity drop: the role of driver characteristics

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ABSTRACT

This paper shows that traffic hysteresis arises due to variable driver characteristics within each driver and has a profound reproducible impact on the periodicity and development of traffic oscillations and the bottleneck discharge rate. Following an oscillation, traffic initially exhibits lower density and flow; then it evolves toward and eventually exceeds the equilibrium, whereupon another oscillation is instigated by an aggressive driver(s) with relatively small response time and minimum spacing. Thereafter, traffic reverts to lower density and flow and repeats the evolutionary cycle. Aggressive driver behavior also leads to hysteresis loops that induce the upstream propagation of oscillations; with larger hysteresis loops inducing larger oscillation growth. Our finding also suggests that the bottleneck discharge rate can diminish by 8%-23% when drivers adopt larger response times in reaction to disturbances. This finding suggests that existing capacity-drop theories, with lane-changes as the main factor, may be incomplete.

Keywords: traffic hysteresis, stop-and-go oscillations, driver behavior, capacity drop
1. INTRODUCTION

Traffic phenomena of capacity drop, stop-and-go traffic oscillations and traffic hysteresis have garnered attention of traffic scientists for many years. Several studies attributed capacity drop, a reduction in bottleneck discharge rate, to systematic lane changes near an active bottleneck that create “voids” in traffic streams due to bounded accelerations (e.g., Elefteriadou et al., 1995; Cassidy and Rudjanakanoknad, 2005; Laval et al., 2005; Laval and Daganzo, 2006; Chung et al., 2007; Patire and Cassidy, 2011). Lane changes are also found to instigate traffic oscillations (Ahn and Cassidy, 2007; Zheng et al., 2011b). However, more recent studies confirm that oscillations can arise by other factors such as instabilities in car-following created by rubbernecking (Laval and Leclercq, 2010; Zheng et al., 2011b; Chen et al., 2012a). Once triggered, oscillations propagate upstream as kinematic waves and often grow in amplitude due to car-following and/or lane-changing behavior (Ahn and Cassidy, 2007; Mauch and Cassidy, 2002). Zheng et al. (2011b) and Chen et al. (2012a) provide a comprehensive review of literature on oscillations.

Several theories have been proposed to explain the mechanism of oscillations’ formation and propagation. From the physical and mathematical perspectives, the formation and propagation of traffic oscillations are attributed to system instability, either linear or non-linear (Gasser et al., 2004; Orosz et al., 2004; Schönhof and Helbing, 2007; Ward and Wilson, 2011; Wilson, 2008; Wilson and Ward, 2011). In contrast, Laval and Leclercq (2010) and Chen et al. (2012a) developed a behavioral car-following model that extends Newell’s car-following model (Newell, 2002) to describe the mechanism. The simulation results showed that their model was able to
reproduce key characteristics of traffic oscillations, including periods, propagations, and
amplitude growth, that are qualitatively consistent with empirical observations.

It is widely believed that traffic hysteresis, characterized by a delayed recovery in speed and flow
as vehicles emerge from a traffic disturbance (Treiterer and Myers, 1974), is inherent to
oscillatory driving. A conventional conjecture is that traffic hysteresis is a result of asymmetry
between acceleration and deceleration characteristics (Newell, 1965; Yeo and Skabardonis,
2009; Zhang, 1999; Zhang and Kim, 2005). Some recent studies attribute it to the heterogeneity
of driver population. Notably, Wong and Wong (Wong and Wong, 2002) proposed a multi-class
kinematic wave model assuming multiple driver classes that follow different flow-density
relationships. Their model was able to generate certain macroscopic traffic hysteresis in
simulations.

Ahn et al. (2013) and Laval (2011) point out that traffic hysteresis has been exaggerated in the
literature because measurements were taken in non-steady conditions that arise during
acceleration/deceleration. They respectively developed microscopic and macroscopic methods
to account for non-steady conditions in measuring the orientation and magnitude of hysteresis.
They found that hysteresis, when properly measured, is less frequent and smaller in magnitude
than previously believed. Furthermore, reverse hysteresis, characterized by higher flows during
acceleration, has been found to be more prevalent than previously believed. Nevertheless, these
studies confirmed common existence of traffic hysteresis in oscillatory traffic.
More recently, Chen et al. (2012b) linked directly driver characteristics (e.g., aggressiveness and the reaction patterns to disturbances) to the hysteresis orientation when encountering traffic oscillations. They found that different development stages of oscillations – growth and fully-developed stages – are associated with different traffic hysteresis orientations (and thus driver characteristics).

Despite significant improvements in our understanding of the aforementioned traffic phenomena, it remains to be seen (i) how periodic oscillations form; (ii) how they transform from localized disturbances to well-developed ones due to driver characteristics (therefore, hysteresis characteristics); and (iii) how driver characteristics contribute to capacity drop. Note that Laval and Leclercq (2010) and Chen et al. (2012a) were able to reproduce (i) and (ii) in their simulations; however, the exact mechanisms remain unclear. The present study investigates these three issues. Note that this study focuses on the mechanisms related to driver car-following since the effects of lane changes on capacity drop and oscillations have been investigated previously.

Notably, this study elucidates traffic evolution leading to periodic oscillations. We found that emerging from an earlier oscillation, traffic state is characterized by lower density and flow (relative to the equilibrium). Then the density-flow relationship evolves toward and eventually exceeds the equilibrium relationship, whereupon another oscillation is instigated by an aggressive driver(s) with relatively small response time and minimum spacing. Thereafter, traffic reverts to lower density and flow and repeats the evolutionary cycle, leading to periodic
oscillations. Furthermore, aggressive drivers with large hysteresis instigate the transitions from localized disturbances to well-developed ones that propagate upstream.

Most importantly, we conjecture that drivers’ reactions to oscillations around a bottleneck have a profound impact on capacity drop. Specifically, a reduction in bottleneck discharge rate ensues – without lane-changes – when drivers become less aggressive and adopt larger response times and minimum spacing. We describe the mechanism and formulate the reduction in terms of driver characteristics parameters. We further conduct a simulation experiment to verify this conjecture and gain further insight into the mechanism. The reduction estimated from the formulation is found to be consistent with the reductions reported in various empirical studies. This finding is particularly noteworthy in light of the fact that previous theories attribute capacity drop to systematic lane-changes.

This paper is organized as follows. Section 2 describes the site and data used in this study. Section 3 describes the method to measure driver characteristics and traffic hysteresis. Section 4 presents the major findings on the development of oscillations in relation to driver characteristics (and therefore traffic hysteresis). Section 5 describes the mechanism of the reduction in bottleneck discharge rate in relation to driver characteristics and supporting evidence based on simulations. Discussions and future research are provided in the end.
2. DATA

This study uses the NGSIM vehicle trajectory data collected on US 101 (NGSIM, 2006); see Figure 1 for the schematic of the study site\(^1\). The trajectories on a 2100-foot segment were extracted from 7:50-8:35 a.m. on June 15, 2005, with the temporal resolution of 0.1 s. During the data collection period, the entire section was congested (speed < 55 ft/s). In the first 12 minutes, several oscillations formed within the study section, evidently induced by the presence of a maintenance crew in the median (observed from the video). The traffic condition gradually worsened, and oscillations from a downstream bottleneck propagated to the study section starting around 8:02 a.m. To study the effects of driver characteristics, nine oscillations that formed within the two left lanes of the study section were analyzed in this study, where lane changes were relatively infrequent. Six of these oscillations were attributable to car-following behavior and transformed into substantial disturbances that propagated upstream; see Figure 1b for several oscillations sampled from the median lane. For the other three, lane-changing apparently had some influence in the formation process. (See Appendix A for all nine cycles analyzed.)

Figure 1

3. MEASUREMENT METHODS

In this section, we describe our methods to characterize and measure driver characteristics and traffic hysteresis.

\(^1\) The other NGSIM site, an eastbound section on I-80, showed instances of oscillations instigated mainly by lane changes (Zheng et al., 2011b).
3.1. Driver Behavior

To characterize driver behavior, we adopt the asymmetric behavioral (AB) car-following model proposed in Laval and Leclercq (2010) and validated and enhanced in Chen et al. (2012a). In this model, the car-following behavior of drivers in congestion is described by:

$$x_i(t) = x_{i-1}(t - \tau_i(t)) - \delta_i(t),$$

(1)

where $x_i(t)$ is the position of driver $i$ at time $t$; $\tau_i(t)$ is the response time of driver $i$ measured at $t$ (i.e., travel time of a kinematic wave from driver $i-1$ to $i$, marking a deceleration/acceleration wave); and $\delta_i(t)$ is the minimum spacing at $t$; see Figure 2 for an illustration. Note that the special case of $\tau_i(t) = \tau$ and $\delta_i(t) = \delta_i$ corresponds to the simplified car-following model by Newell (Newell, 2002). Assuming a constant wave speed of $w$, $\tau_i(t)$ and $\delta_i(t)$ are respectively expressed as $\eta_i(t)\tau$ and $\eta_i(t)\delta$, where $\tau$ and $\delta$ are the time-independent equilibrium values derived from the Kinematic Wave (KW) model (Lighthill and Whitham, 1955; Richards, 1956) with a triangular shape. Thus, $\tau = -\frac{1}{wk}$ and $\delta = \frac{1}{k}$, where $k$ represents the jam density. Then, $\eta_i(t)$ is the ratio of the actual response time (or jam spacing) to the equilibrium; i.e.,

$$\eta_i(t) = \tau_i(t)/\tau,$$

(2)

The evolution of $\eta_i(t)$ describes the time-dependent driver characteristics (relative to the equilibrium).

Figure 2

As illustrated in Chen et al. (2012a; 2012b), $\eta_i(t)$ is relatively constant over time before a driver encounters a stop-and-go disturbance but deviates as the driver decelerates and accelerates, reflecting her/his reaction pattern. The driver characteristics during an oscillation are measured in three key elements: driver category, reaction pattern, and response timing. Table 1

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summarizes the ways in which these elements are measured and categorized. Note that if
\( \eta_i(t) < 0.9 \) before an oscillation, the actual trajectory is closer to the leader’s trajectory, in which
case the driver is categorized as originally aggressive (OA). In contrast, \( \eta_i(t) > 1.1 \) represents an
originally timid (OT) driver. \( \eta_i(t) \) in between represents an originally Newell (ON) driver,
displaying approximately equilibrium behavior. For the driver reaction pattern, a concave or
non-decreasing evolution of \( \eta_i(t) \) during an oscillation indicates that the driver adopts less
aggressive behavior at least momentarily, whereas a convex evolution represents adopting more
aggressive behavior. Finally, based on the timing that the driver adopts more or less aggressive
behavior, the behavior is categorized as early or late response. These criteria to categorize driver
types will be used throughout this study.

Table 1

In our analysis, the wave speed is fixed to be -10 mph based on earlier findings (e.g., Laval and
Leclercq, 2010; Durent et al., 2011; Chen et al., 2012a). For \( \tau \), we take the average of \( \tau_i(t) \) in
equilibrium across all drivers sampled.

3.2. Hysteresis

We now present the ways in which traffic hysteresis is measured at the micro and macro levels.
Microscopic measurements are taken from the evolutions of speed vs. steady-state spacing (Ahn
et al., 2012) and speed vs. \( \eta \) relations (Chen et al., 2012b) of individual vehicles. As illustrated
in Figure 3, both methods account for non-steady conditions via steady-spacing, \( s_i(t) \), and \( \eta_i(t) \),
as given by:

\[
s_i(t) = (v_{i-1}(t - \tau_i(t)) - w)\tau_i(t) \tag{3}
\]
\begin{equation}
\eta_i(t) = \frac{s_i(t)}{s(v_{i-1}(t-\tau_i(t)))} = \frac{(v_{i-1}(t-\tau_i(t))-w)\tau_i(t)}{(v_{i-1}(t-\tau_i(t))-w)\tau} = \frac{\tau_i(t)}{\tau}
\end{equation}

Note that $\eta_i(t)$ represents the ratio of driver $i$’s steady-state spacing to equilibrium steady-state spacing, $S\left(v_{i-1}(t-\tau_i(t))\right)$, assuming $\tau$ and $\delta$, where $S(\cdot)$ is the equilibrium spacing function.

Figure 3

Traffic hysteresis is categorized as nominal, ordinary, and reverse based on the orientation of the $v$-$s$ and $v$-$\eta$ evolutions. The \textit{nominal} hysteresis is characterized by the evolution of $v$-$s$ relations in the same (linear) path during deceleration and acceleration (Figure 4(a)); i.e., a vehicle trajectory is nearly superimposed with the theoretical trajectory described by Newell’s model (Figure 4(b)). For \textit{ordinary} hysteresis, as famously observed by Treiterer and Myers (1974), $v$-$s$ relations evolve clockwise (CW), which typically results in a trajectory below the Newell trajectory; see Figure 4(c)-(d). Finally, \textit{reverse} hysteresis is characterized by counterclockwise (CCW) $v$-$s$ evolutions, resulting in a trajectory above the Newell trajectory; see Figure 4(e)-(f). It turns out that the hysteresis orientation is consistent in $v$-$\eta$ relations with different advantages that $\eta$ is independent of speed, while $s$ allows for a more straightforward physical interpretation; see Figure 5(a)-(b) for an example.

Figure 4

Figure 5

The magnitude is measured in terms of average differences in $s$ and $\eta$ (i.e., $\Delta s$ and $\Delta \eta$), respectively), between acceleration and deceleration branches over the span of observed speed; see Figure 5(a)-(b). Notably, $\Delta s \gg 0$ suggests a CW loop (thus ordinary hysteresis), and $\Delta s \ll 0$ for a CCW loop (thus reverse hysteresis). Note that this method can under-estimate the
magnitude of hysteresis if multiple loops are present; however, single loop hysteresis was predominant in our study.

At the macro level, we measure the density-flow evolution along the movement of a vehicle platoon according to Edie’s generalized definitions (Edie, 1961; Laval, 2011); see Figure 5(c). Each measurement region is shaped a parallelogram with the incline of the average wave speed, \( w \), to ensure a near-steady state inside the region. The hysteresis orientation is conveniently consistent with the orientation at the micro level; a CW (CCW) evolution of density-flow represents ordinary (reverse) hysteresis. The magnitude of macroscopic hysteresis can then be measured as the average flow difference over the density range; see Figure 5(d).

4. DEVELOPMENT OF OSCILLATIONS

A cycle of an oscillation is marked by deceleration succeeded by acceleration. In congestion, an oscillation propagates in space as a set of backward-moving deceleration and acceleration waves; see Figure 6. The origins (i.e., formation) and propagations of these waves are identified by continuous wavelet transform, a time-frequency spectral analysis method that can systematically detect sudden changes in speed. The readers are referred to (Zheng et al., 2011a; Zheng et al., 2011b) for the details of this method. Essentially, we used this method to identify the first vehicle that exhibited significant changes in speed (marking a disturbance formation). We attribute the formation to lane-changing if there was a lane change immediately ahead of that vehicle. Otherwise, we consider car-following behavior as the instigator. Nine oscillation cycles
are analyzed in this study. Of these oscillations, six formed spontaneously due to rubbernecking around a maintenance crew present in the median (Chen et al., 2012a)\(^2\). As noticeable in Figure 6, these oscillations underwent different development stages as they propagated in space. Four distinct stages are characterized below:

(i) Precursor: The speeds of the deceleration and acceleration waves are close to zero, indicating localized slow-and-go driving motions.

(ii) Growth: The waves propagate backward in space at the speed of 10-15 mph, and the minimum speed of vehicles (a measure of oscillation amplitude) decreases significantly as the waves propagate.

(iii) Stable: The amplitude remains relatively constant as waves propagate backward in space.

(iv) Decay: The amplitude diminishes as waves propagate (not shown in Figure 6).

Figure 6

In the following section, we will focus on stages (i)-(iii) since (iv) was rarely observed in our sample. We analyze the effects of driver behavior and hysteresis on oscillations formation, transition (from the precursor to the growth stages), and amplitude development.

\(^2\) The remaining three instances were potentially attributable to lane-changing and thus, excluded in the analysis of formations.
4.1. Oscillation Formation

In this section, we present the mechanism of spontaneous formation of oscillations by car-following behavior and the periodicity of the formations. Figure 7 illustrates how the traffic condition, measured by density-flow relationship, develops over time around the origin of a stop-and-go (SG) disturbance. More precisely, we measure the density-flow relations (according to Edie’s definitions) for a platoon of 5-6 vehicles over a short distance (e.g., 300 ft; see Figure 7(a)) and observe how the relationship changes over successive platoons. It is evident from Figure 7(b) that each of platoons 1-3 exhibits relatively stable density-flow relationship, judging by small clusters of points. Moreover, emerging from an earlier oscillation, the traffic state lies well below the equilibrium fundamental diagram (FD) and gradually recovers toward the equilibrium; see Figure 7(b). A SG disturbance forms in platoon 4 when the density-flow relationship significantly exceeds the equilibrium FD. After the formation, vehicle platoons 4-5 exhibit significant traffic hysteresis. In summary, the evolution of traffic condition around a SG disturbance formation obeys the following sequence: (i) low density and flow relative to the equilibrium \( \rightarrow \) (ii) recovery to equilibrium \( \rightarrow \) (iii) high density and flow \( \rightarrow \) (iv) SG disturbance formation \( \rightarrow \) (v) back to (i).

The result suggests that the drivers around the formation of a SG disturbance are more aggressive than the average, leading to higher density-flow relations; i.e., their \( \eta_l^0 \) values are smaller. Figure 8 shows the \( \eta_l^0 \) values for the vehicles in platoons 3-5 and further confirms this observation; several vehicles prior to the formation are characterized by mostly OA (and some ON) drivers (see Table 1 again for the driver category). This observation was consistent for the six cycles of oscillations that spontaneously formed.
4.2. Oscillation Transition and Traffic Hysteresis

As Figure 6 illustrates, the transition into the growth stage is characterized by a marked change in wave propagation. Thus, the transition point can be identified as the breakpoint between two trend lines of wave propagations. (The breakpoints can be identified systematically using the Bottom-Up algorithm as in (Zheng et al., 2011b). However, the transition points were rather evident to the naked eye for the most cycles analyzed.) Occasionally, the transitions occurred with different vehicles during deceleration and acceleration. Thus, we differentiate transitions in deceleration and acceleration waves. Of the nine cycles of oscillations analyzed, the transition occurred without lane-changing in seven cases.

Table 2 summarizes the characteristics of the transition vehicles for the seven cycles. We find that the transition drivers are mostly OA or ON with the exception of one case of OT. Most of these drivers adopt concave or non-decreasing reaction patterns with late responses, resulting in CW hysteresis. The transition drivers exhibit large magnitude of hysteresis (measured by $\Delta \eta$), corresponding to 0.367 and 0.336 on average along deceleration and acceleration waves, respectively, as compared to the overall average of 0.06. Finally, it is also notable that most transition driver exhibit much larger CW hysteresis loops than the drivers in any other stages.

Table 2
4.3. Oscillation Growth and Traffic Hysteresis

We now turn our attention to oscillation growth in relation to driver behavior and traffic hysteresis. We find that different stages of traffic oscillations are characterized by different hysteresis loops. One can see from Figure 9 that the vehicle platoons in precursor and growth stages exhibit significant CW hysteresis loops, whereas the stable stage exhibits negligible hysteresis. These findings are corroborated using the microscopic measurements; see Table 3. The hysteresis magnitude is the largest (and significant) in the precursor stage, followed by the growth stage, but is negligible in the stable stage.

As in Chen et al. (2012b), we find that the large hysteresis in the precursor and growth stages is attributable to the driver category and reaction patterns. Specifically, OA and ON drivers often adopt non-decreasing or concave reaction patterns with late responses that lead to large CW hysteresis; see Figure 10 for an example. These reaction patterns are more prevalent in the precursor and growth stages as shown in Chen et al. (2012b) and confirmed in this study.

In this study, we further unveil the mechanism of oscillation growth; see Figure 11. We observe that when an OA or ON driver adopts any of the above reaction patterns, (s)he either (i) coasts and delays the acceleration process (see $\bar{x}_i(t)$ in Figure 11) or (ii) further reduces the speed (i.e., $\Delta v_{min} < 0$) and amplifies the SG disturbance (see $\bar{x}_i(t)$ in Figure 11). (Figure 10 is an example of the latter scenario.) The latter makes up approximately 65% of the oscillation growth
cases. Notably, both scenarios would result in CW hysteresis; however, scenario (ii) would result in larger magnitude of hysteresis than scenario (i). Furthermore, the larger the reduction in minimum speed is, the larger the hysteresis magnitude is. This is further corroborated in Figure 12, which shows strong and statistically significant correlations between $\Delta v_{\text{min}}$ and $\Delta s$, as well as $v_{\text{min}}$ and $\Delta \eta$ ($p$-values of $2.29 \times 10^{-13}$ and $6.50 \times 10^{-13}$, respectively).

Figure 11
Figure 12

5. EFFECT OF DRIVER REACTIONS ON BOTTLENECK DISCHARGE RATE

In the previous section, we showed that driver category and reaction pattern affect oscillations formations and development. In this section, we study the impact of driver characteristics on the capacity drop phenomenon. We describe the mechanism, formulate the impact, and corroborate our conjecture through simulations.

5.1. Formulation of Reduction in Bottleneck Discharge Rate

We conjecture that driver reaction patterns can lead to a reduction in bottleneck discharge rate. Figure 13 illustrates four possible cases in which driver reaction patterns, following a disturbance, can affect the bottleneck discharge rate. A disturbance can be created by any factor including instability in car-following, rubber-necking, and lane-changing. Case 1 represents a driver with a non-decreasing reaction patterns (see Figure 13(a)) in which a driver adopts a larger response time ($\tau_i(t)$) and minimum spacing ($\delta_i(t)$) following a disturbance, resulting in $\eta^1_i > \eta^0_i$ and a CW hysteresis loop. Note that a concave reaction pattern (dashed line in Figure 13(a)) is very rare around the active bottleneck because the lead vehicle is traveling at the free-flow speed,
$v_f$, and thus recovering $\eta_i^0$ would be very difficult. Therefore, in many cases, drivers who would have otherwise adopted concave reaction patterns would instead display non-decreasing patterns. For this reason, we treat both patterns together as case 1. Figure 13(b) shows a (hypothetical) trajectory for case 1 (labeled ‘1’), which lies beneath the Newell trajectory (dashed trajectory), indicating additional delay and a lower discharge rate.

In case 2, a driver exhibits a convex reaction pattern but adopts larger $\tau$ and $\delta$ in the end ($\eta_i^1 > \eta_i^0$). As illustrated in Figure 13(b), a driver of this type would also experience additional delay. In contrast, if the driver adopts $\eta_i^1 = \eta_i^0$ (case 3) or $\eta_i^1 < \eta_i^0$ (case 4), there would be no additional delay. In fact, a higher discharge rate can be achieved in case 4 due to the smaller headway. This behavior, however, was very rare in our observation, indicating that a reduction in bottleneck discharge rate is more probable in reality. Below, we derive a formulation to estimate the reduction in bottleneck discharge rate due to different driver reaction patterns.

In the absence of a change in driver behavior (i.e., constant $\eta$), the bottleneck discharge rate, $q_{bn}^{NW}$, can be expressed as the reciprocal of average headway across drivers:

$$q_{bn}^{NW} = \frac{n}{\sum^N_i \left( \tau_i + \frac{L_w}{v_f} \right)} = \frac{n}{\sum^N_i \eta_i^2 \left( \frac{w+v_f}{v_f} \right)} = \frac{v_f}{(w+v_f)\tau} \cdot \frac{n}{\sum^N_i \eta_i^0} \quad \text{(5)}$$

where $n$ represents the number of vehicles in the measurement period. Note that this represents the bottleneck discharge rate assuming Newell’s car-following model. With different reaction patterns (i.e., variable $\eta$), the bottleneck discharge rate assuming the AB model, $q_{bn}^{AB}$, is:

$$q_{bn}^{AB} = \frac{n}{\sum^N_i \eta_i^1 \left( \frac{w+v_f}{v_f} \right)} = \frac{v_f}{(w+v_f)\tau} \cdot \frac{n}{\sum^N_i \eta_i^1} \quad \text{(6)}$$
Thus, the change in bottleneck discharge is:

\[ q_{bn}^{AB} - q_{bn}^{NW} = \frac{v_f}{(w+v_f)\tau} \left( \frac{n}{\Sigma \eta_i^1} - \frac{n}{\Sigma \eta_i^0} \right) \]  

(7)

Recall that we calibrated the fundamental diagram using the mean of equilibrium uniformity values \((\bar{\eta})\) (before oscillations) across all sampled vehicles; the y-intercept of the congested branch equals \(\frac{1}{\tau}\), and the x-intercept equals to \(\frac{1}{\delta}\). Thus the mean of \(\eta_i^0 \Rightarrow \eta^0 = \frac{\Sigma \eta_i^0}{n}\) equals to 1, and \(\eta_i(t)\) reflects the behavior of an individual driver with respect to the calibrated fundamental diagram.

Then, the mean of \(\eta_i^1 \Rightarrow \eta^1 = \frac{\Sigma \eta_i^1}{n}\) different from 1 represents a change in the fundamental diagram; the y-intercept and x-intercept respectively equal to \(\frac{1}{\eta^1 \tau}\) and \(\frac{1}{\eta^1 \delta}\) in the modified fundamental diagram. The difference, \(\frac{1}{\eta^1} - \frac{1}{\eta^0}\), represents the change in both y- and x-intercepts, which is proportional to the change in the bottleneck discharge flow (see equation (7)).

If the proportions of drivers with different reaction patterns are known, the reduction in bottleneck discharge rate can also be written as:

\[ \Delta q = q_{bn}^{AB} - q_{bn}^{NW} = \frac{v_f}{(w+v_f)\tau} \left( \frac{1}{\Sigma \eta_k \bar{\eta}^0_k} - \frac{1}{\Sigma \eta_k \bar{\eta}^1_k} \right) \]  

(8)

where \(k \in \{1,2,3,4\}\), denoting different reaction pattern groups \((k = \{1,2,3\}\) for the three patterns, non-decreasing, concave, and convex, and \(k = 4\) for all other (mostly constant) patterns); \(p_k\) represents the proportion of drivers in reaction group \(k\); \(\bar{\eta}^0_k\) and \(\bar{\eta}^1_k\) respectively denote average \(\eta^0\) and \(\eta^1\) within each reaction group. Finally, the percent reduction in bottleneck discharge rate is:

\[ \frac{q_{bn}^{AB} - q_{bn}^{NW}}{q_{bn}^{NW}} = \frac{\Sigma p_k \bar{\eta}^0_k}{\Sigma p_k \bar{\eta}^1_k} - 1 \]  

(9)
Table 4 shows the estimated values of $p_k$, $\eta_k^0$, and $\eta_k^1$ for the three reaction pattern groups observed from the NGSIM dataset. Thus, this particular composition of driver reaction patterns would result in the overall reduction of 16%, which is comparable to the values reported in various empirical studies (e.g., Cassidy and Bertini, 1999; Bertini and Leal, 2005). The effect tends to wane as oscillations develop: the reduction is the largest during the precursor stage (27%), followed by the growth stage (18%), and then the stable stage (7%). Therefore, the time (or the number of vehicles) taken until oscillations become well-developed and stable is also a critical factor for the overall reduction in bottleneck discharge rate. The drivers with concave reaction patterns are probably over-represented in Table 4 because the NGSIM data were collected in congestion. For an active bottleneck, we expect a higher proportion of non-decreasing pattern in lieu of concave pattern. Furthermore, we expect $\eta_k^1$ to be larger for the concave reaction pattern since it would be difficult for drivers to recover $\eta_k^0$ once they deviate.

Table 4

5.2. Simulation Experiment

To understand the impact of driver reactions on the bottleneck discharge rate, we conduct a simulation experiment. As in Chen et al. (2012a), we assume a bottleneck created by rubbernecking. In the experiment, we simulate a one-lane freeway segment that is 4.39 mi long with a rubbernecking zone located between 1.86 and 1.90 mi; see Figure 14(a). Traffic demand is set to be the freeway capacity. A driver traveling at speed $v$ has probability $r$ to reduce his speed by a proportion $\Delta v$ when he enters the rubbernecking zone.
The AB model can be described with five driver-specific parameters \([\eta_i^0, \eta_i^T, \eta_i^1, \epsilon_i, s_i]\), where \(\epsilon_i = \epsilon_i^0 = \epsilon_i^1\) since \(\epsilon_i^0\) and \(\epsilon_i^1\) are not significantly different (Chen et al., 2012a), and \(s_i\) is binary variable, either early or late, for the timing of the change in \(\eta\). In Newell’s simplified car-following model, only one driver-specific parameter, \(\eta_i^0\), is needed to describe the car-following behavior.

For the simulation, the model parameters are selected from a pool of samples that are extracted from the vehicle trajectory data for lane 1 of US 101 in the period of 7:50am-8:05am. We take different samples for different stages of oscillations development to incorporate the variations in driver reaction patterns and timing. However, to maximize the sample size, we consider two stages (rather than four), pre-stable and stable stages. Since driver behaviors in the precursor and growth stages are similar, the two stages are together treated as the pre-stable stage. The decay stage is not considered because the mechanism of this stage is still unclear and it is not the focus of this study. A total of 56 samples were drawn for each stage.

For selecting parameter values, the sample enumeration method (Ben-Akiva and Lerman, 1985) is used to preserve potential correlations between model parameters. Specifically, each time a vehicle is generated, a \(\eta_i^0\) value is selected from the combined pool of samples across the two stages because \(\eta_i^0\) is independent of the oscillation stage. For other parameters of the AB model, we first determine the oscillation stage that the vehicle is in using a vehicle-foreseeing approach (see Appendix B for details) and identify the corresponding pool of samples. Parameter values are then drawn from the subsample of the pool that consists of similar \(\eta_i^0\) values (\(\eta_i^0 \pm 0.05\)).
5.3. Simulation results

Examples of simulated trajectories are shown in Figure 14; see (b) for a macroscopic view over 30 min and (c) for the details of the oscillation circled in (b). One can see that the AB model is able to reproduce the precursor, growth, and stable stages. Moreover, each stage exhibits traffic hysteresis that is consistent with empirical observation: vehicles in the pre-stable stage exhibit CW hysteresis loops (Figure 14(c)-(d)) while the hysteresis in the stable stage is nearly negligible (Figure 14(e)-(f)).

We also quantify the effect of variable driver reaction patterns on the bottleneck discharge rate. To do this, we run simulations using Newell’s simplified car-following model to measure the baseline bottleneck discharge rate and then using the AB model to measure the effect of variable driver reactions on the bottleneck discharge rates. Then, the difference between the two simulation results represents the effect of variable driver reactions on the bottleneck discharge rate, $\Delta q$.

Figure 15(a)-(b) respectively show the discharge rates, $q_{bn}^{NW}$ and $q_{bn}^{AB}$, measured at 4.39 mi, which is 2.49 mi downstream of the rubbernecking zone. As expected, both $r$ and $\Delta v$ have negative effects on the discharge rates. More importantly, the discharge rates are substantially lower when variable driver reactions are considered (see Figure 15(b)); the reductions in discharge rates range from 8% to 23% (see Figure 15(c)). Notice that this is comparable to the 16% reduction obtained from Equation (9) and supports our conjecture that variable driver reactions significantly affect the bottleneck discharge rate.
Parameters $r$ and $\Delta v$ also negatively affect $\Delta q$; see Figure 15(c). The results are rather intuitive. As $r$ increases, there is a greater portion of drivers rubbernecking, and as a result, rubbernecking behavior dominates the bottleneck discharge rate change. With a larger speed drop, $\Delta v$, during an oscillation, it takes less time and fewer vehicles to reach the stable stage. Therefore, the effect of driver reactions quickly wanes, resulting in a smaller overall impact on $\Delta q$.

6. CONCLUSIONS

This study has investigated the mechanisms of traffic oscillations development and capacity drop in relation to driver characteristics manifested in traffic hysteresis. Empirical observations showed that aggressive drivers were responsible for the spontaneous formations of oscillations that were analyzed in this paper. Emerging from an oscillation, traffic initially exhibited lower than equilibrium flows and densities and then gradually recovered toward equilibrium. Successive oscillations were instigated by aggressive drivers after recovering equilibrium. This finding may shed some light for explaining the periodic formations of traffic oscillations: the period of oscillations may be related to the time it takes to recover equilibrium conditions.

Once formed, oscillations underwent four development stages: precursor, growth, stable, and decay stages, although the decay stage was seldom observed in our data due to the limited extension of our study site. We found that the transition from the precursor to growth stages occurred when relatively aggressive drivers exhibited large hysteresis by adopting larger response times and minimum spacing. Furthermore, similar behavior contributed to significant growth in oscillations amplitude in the precursor and growth stages. Statistical results further confirmed high correlation between growth in oscillations amplitude and hysteresis magnitude. This study established the links explicitly between driver characteristics and reactions to
disturbances, traffic hysteresis, and oscillations development (including formations) and quantified the relations.

This study also found that bottleneck discharge rates can diminish when drivers change their response time and minimum spacing in reaction to disturbances around a bottleneck. We described the mechanism and quantified the reduction in this paper, which were verified through a simulation experiment. This important finding underscores that “capacity drop” can ensue, without lane-changes, due to variable driver car-following behavior. This finding suggests that existing capacity-drop theories are incomplete and should account for variable driver characteristics within each driver.

Note that this study focused on the car-following effect without considering the lane-changing behavior. Further research is needed to better understand the contribution of each effect, the dominant effect, and the interactions between these behaviors in the development of periodic oscillations and bottleneck discharge rates. In particular, it is possible that lane changes may have an indirect effect by creating conditions more prone to oscillations’ formations. This potential indirect effect of lane changes (and car-following for that matter) was not investigated in this study. These issues are being investigated by the authors. Finally, this study was conducted on a single freeway site due to limited availability of suitable trajectory data. Confirmation of the findings reported in this study is left for future research.
ACKNOWLEDGEMENT

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REFERENCES


APPENDIX A: SAMPLED OSCILLATIONS

Figure A1
APPENDIX B

In simulating a vehicle, one needs to first determine the oscillation stage that the vehicle will encounter to generate parameters from an appropriate pool of samples. This is challenging because in simulations the vehicle position is updated at each time tick based on the information (position and speed) of the vehicle immediately ahead in the previous time tick. To reduce numerical errors, a small time tick is used, typically in the order of $\tau \approx 1.71$ s. Since deceleration normally takes much longer, one essentially needs to predict if the lead vehicle will eventually come to a complete stop and enter the stable stage.

To address this problem, the development stage for vehicle $i$ is determined based on the speed of $n$ vehicles ahead when it encounters a deceleration wave (at time $t$ in Figure A1). If any of the lead vehicles has reached zero speed, vehicle $i$ is assumed to be in the stable stage; otherwise in the pre-stable stage. The value of $n$ is determined by searing the largest vehicle number (i.e., the closest leader to vehicle $i$) that had sufficient time to decelerate to zero speed by $t$. This is achieved by finding $n$, such that the trip time of a deceleration wave from vehicle $i - n$ to vehicle $i$ is approximately equal to the typical duration of deceleration to zero speed ($\approx 20$ s).

Based on examination of the NGSIM data, the $n$ value of 12 proved very reasonable.

Figure B1
### Table 1: Key elements of the behavioral car-following model in (Chen et al. 2012a; 2012b)

<table>
<thead>
<tr>
<th>Element</th>
<th>Variable</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driver category</td>
<td>Constant $\eta_i(t)$ value, $\eta_i^0$, before oscillation</td>
<td>Originally aggressive (OA): $\eta_i^0 &lt; 0.9$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Originally Newell (ON): $0.9 \leq \eta_i^0 \leq 1.1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Originally timid (OT): $\eta_i^0 &gt; 1.1$</td>
</tr>
<tr>
<td>Reaction pattern</td>
<td>Shape of $\eta_i(t)$ during an oscillation</td>
<td>Concave</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Convex</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-decreasing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constant</td>
</tr>
<tr>
<td>Response timing</td>
<td>Starting time of deviation from equilibrium $\eta_i(t)$</td>
<td>Early response: deviates from equilibrium near the beginning of deceleration;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Late response: deviates from equilibrium near the beginning of acceleration</td>
</tr>
</tbody>
</table>
Table 2 Hysteresis magnitude (measured by $\Delta \eta$) of transition vehicles

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Driver category ($\eta^*_i$)</th>
<th>Driver reaction pattern</th>
<th>$\Delta \eta$</th>
<th>Driver category ($\eta^*_j$)</th>
<th>Driver reaction pattern</th>
<th>$\Delta \eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle 1</td>
<td>ON (1.00)</td>
<td>non-decreasing</td>
<td>0.619</td>
<td>ON (1.00)</td>
<td>non-decreasing</td>
<td>0.619</td>
</tr>
<tr>
<td>Cycle 2</td>
<td>ON (0.90)</td>
<td>non-decreasing</td>
<td>0.556</td>
<td>ON (0.90)</td>
<td>non-decreasing</td>
<td>0.556</td>
</tr>
<tr>
<td>Cycle 3</td>
<td>ON (1.05)</td>
<td>concave</td>
<td>0.209</td>
<td>OA (0.50)</td>
<td>non-decreasing</td>
<td>0.112</td>
</tr>
<tr>
<td>Cycle 4</td>
<td>ON (0.90)</td>
<td>non-decreasing</td>
<td>0.387</td>
<td>OT (1.30)</td>
<td>convex</td>
<td>0.318</td>
</tr>
<tr>
<td>Cycle 5</td>
<td>OA (0.75)</td>
<td>concave</td>
<td>0.342</td>
<td>OA (0.50)</td>
<td>concave</td>
<td>0.557</td>
</tr>
<tr>
<td>Cycle 6</td>
<td>OA (0.60)</td>
<td>concave</td>
<td>0.206</td>
<td>OA (0.60)</td>
<td>concave</td>
<td>0.206</td>
</tr>
<tr>
<td>Cycle 9</td>
<td>ON (1.10)</td>
<td>concave</td>
<td>0.251</td>
<td>ON (1.00)</td>
<td>convex</td>
<td>-0.019</td>
</tr>
</tbody>
</table>

**Average**

<table>
<thead>
<tr>
<th></th>
<th>Deceleration</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition</td>
<td>0.367</td>
<td>0.336</td>
</tr>
<tr>
<td>All</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Precursor stage</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Growth stage</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Stable stage</td>
<td>-0.01</td>
<td></td>
</tr>
</tbody>
</table>
Table 3  Summary of measured amplitudes of oscillations and hysteresis

<table>
<thead>
<tr>
<th>Oscillation Stage</th>
<th>Sample Size (vehs)</th>
<th>Average $\Delta v_{\text{min}}$ (ft/s)</th>
<th>Average $\Delta s$ (ft)</th>
<th>Average $\Delta \eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precursor</td>
<td>80$^1$ (64$^2$)</td>
<td>-2.07</td>
<td>20.04</td>
<td>0.23</td>
</tr>
<tr>
<td>Growth</td>
<td>58 (77)</td>
<td>-1.46</td>
<td>17.35</td>
<td>0.31</td>
</tr>
<tr>
<td>Stabilization</td>
<td>116 (113)</td>
<td>-0.19</td>
<td>7.59</td>
<td>0.15</td>
</tr>
</tbody>
</table>

$^1$ the sample size based on the oscillation stages determined by deceleration waves

$^2$ the sample size based on the oscillation stages determined by acceleration waves
Table 4  Summary of $p_k$, $\bar{\eta}_k^0$, and $\bar{\eta}_k^1$

<table>
<thead>
<tr>
<th>Stage</th>
<th>Row Labels</th>
<th>$\bar{\eta}_k^0$</th>
<th>$\bar{\eta}_k^1$</th>
<th>$p_k$</th>
<th>$q_{bn} - q_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>precursor (39)</td>
<td>Non-decreasing</td>
<td>0.9</td>
<td>1.47</td>
<td>41%</td>
<td>-39%</td>
</tr>
<tr>
<td></td>
<td>Concave</td>
<td>0.8</td>
<td>0.88</td>
<td>21%</td>
<td>-9%</td>
</tr>
<tr>
<td></td>
<td>Convex</td>
<td>1.21</td>
<td>1.52</td>
<td>36%</td>
<td>-21%</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>1.37</td>
<td>1.37</td>
<td>3%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Overall</td>
<td>1.002</td>
<td>1.366</td>
<td>100%</td>
<td>-27%</td>
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<tr>
<td>growth (24)</td>
<td>Non-decreasing</td>
<td>0.83</td>
<td>1.18</td>
<td>21%</td>
<td>-29%</td>
</tr>
<tr>
<td></td>
<td>Concave</td>
<td>0.81</td>
<td>0.98</td>
<td>29%</td>
<td>-17%</td>
</tr>
<tr>
<td></td>
<td>Convex</td>
<td>1.2</td>
<td>1.41</td>
<td>46%</td>
<td>-14%</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>0.74</td>
<td>0.74</td>
<td>4%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Overall</td>
<td>0.99</td>
<td>1.22</td>
<td>100%</td>
<td>-18%</td>
</tr>
<tr>
<td>stable (57)</td>
<td>Non-decreasing</td>
<td>0.91</td>
<td>1.32</td>
<td>9%</td>
<td>-31%</td>
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<tr>
<td></td>
<td>Concave</td>
<td>0.91</td>
<td>0.96</td>
<td>65%</td>
<td>-5%</td>
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<tr>
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<td>Convex</td>
<td>1.32</td>
<td>1.35</td>
<td>19%</td>
<td>-2%</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>1.07</td>
<td>1.14</td>
<td>7%</td>
<td>-6%</td>
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<tr>
<td></td>
<td>Overall</td>
<td>1</td>
<td>1.11</td>
<td>100%</td>
<td>-7%</td>
</tr>
</tbody>
</table>
Figure 1 (a) Schematic of southbound US 101; (b) Oscillations sampled from the median lane
Figure 2 Measurement of $\tau_i(t)$. 
Figure 3 Measurement of steady-state spacing and $\eta_i(t)$
Figure 4 Examples of traffic hysteresis in speed-spacing relations and trajectory:
(a-b) nominal hysteresis (veh ID 137); (c-d) ordinary hysteresis (veh ID 134); (e-f) reverse hysteresis (veh ID 2184).
Figure 5 Hysteresis measurement: (a) microscopic measurement using speed-spacing relations (veh ID 134); (b) microscopic measurement using speed-spacing relations (veh ID 134); (c) macroscopic measurement based on a platoon; (d) macroscopic measurement using density-flow relations.
Figure 6 Propagation of an oscillation; red (purple) circles mark the deceleration (acceleration) wave.
Figure 7  Traffic evolution around the formation of a traffic oscillation: (a) vehicle trajectories; (b) flow-density evolutions of platoons 1-5 (a rectangle denotes a beginning state and a diamond denotes an ending state).
Figure 8 Driver category around oscillation formation
Figure 9 Examples of platoons in different oscillation stages and the corresponding traffic hysteresis in density-flow relations: (a) precursor stage; (b) growth stage; (c) stable stage; An open circle denotes a starting point; an open square denotes a ending point; a dashed triangle represents a theoretical fundamental diagram with $u=72$ mph, $k_j=267$ veh/mile, and $w=−9.88$ mph.
Figure 10 Empirical example of hysteresis formation (NGSIM, US101)
Figure 11 Analytical illustration of the formation of CW hysteresis loop
Figure 12 Relationship between oscillation growth and hysteresis magnitude: (a) $\Delta v_{\text{min}}$ vs. $\Delta s$; (b) $\Delta v_{\text{min}}$ vs. $\Delta \eta$; $\rho$ represents the Pearson correlation coefficient.
Figure 13 Illustration of the effects of driver reaction patterns on the bottleneck discharge rate: (a) four cases of driver reaction patterns; (b) vehicle trajectories corresponding to four cases of driver reaction patterns.
Figure 14 Simulation results: (a) schematic of the simulated segment; (b) snapshot (from 1.49 mi to 2.36 mi) of simulated trajectories ($a_m=4.92\ ft/s^2$, $p=0.6$, $r=2\%$); (c-d) hysteresis in pre-stable stage; (e-f) hysteresis in stable stage.
Figure 15 Parameter impacts on the bottleneck discharge rate ($a_m=4.92$ ft/s²): (a) baseline (Newell’s model); (b) variable driver reactions (AB model); (c) drop in the discharge rate.
(a) Cycle 1 (US101, Lane 1): Instigated by car-following

(b) Cycle 2 (US101, Lane 1): Instigated by car-following

(c) Cycle 3 (US101, Lane 1): Instigated by car-following
(d) Cycle 4 (US101, Lane 1): Instigated by car-following

(e) Cycle 5 (US101, Lane 1): Instigated by car-following or lane-changing

(f) Cycle 6 (US101, Lane 2): Instigated by car-following
(g) Cycle 7 (US101, Lane 2): Instigated by car-following or lane-changing

(h) Cycle 8 (US101, Lane 2): Instigated by car-following

(i) Cycle 9 (US101, Lane 2): Instigated by lane-changing

Figure A1 Sampled oscillations
Figure B1 Illustration of determining the stage of oscillation development